SECOND ORDER TAYLOR POLYNOMIAL APPROXIMATION OF TRIPLE INTEGRAL

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Abstract

Numerical integration is very important due to its wide usage in many areas such as Applied Mathematics, Physics, Computational Sciences and Engineering. Finding precise solution to real-world problems those involving definite triple integrations is very much complicated and in many occasions it is even impossible. To handle this situation, computational researchers work with approximation methods. Meantime, computational scientists work to find or improve existing methods in order to solve the problem more accurately in short time. In literature, there are numerous digital integrative approaches discussed but only few papers dealt with approximation of triple integral. In this paper we suggest an approximation technique for triple integration that uses second order Taylor polynomial. We consider a function $f(x, y, z)$ of three variables defined on a closed region $R = [a, b] \times [c, d] \times [e, f]$. Then, we divide the region R into sub-regions as the interval $[a, b]$ into l sub-rectangles$[x_i,\ x_{i+1}]$ of equal width $\Delta x = \frac{b-a}{l}$, the interval $[c, d]$ into m sub-interval $[y_j,\ y_{j+1}]$ of equal width $\Delta y = \frac{d-c}{m}$ and the interval$[e, f]$ in to n sub-intervals $[z_k,\ z_{k+1}]$ of equal width $\Delta z = \frac{e-f}{n}$. We then use Taylor polynomial of degree two to find an approximation formula to approximate triple integrals by considering the given function f over each sub-interval by choosing the middle point of each interval. We applied it to evaluate some selected known algebraic, trigonometric, exponential and mixed functions. The results are compared with mid-point rule and the exact value and observed that generally we achieved the solution much quicker and with least error. Only in the case of trigonometric function, we got higher error for unique dimensions however the accuracy increases with dimensions.

Keywords: Numerical integration, Taylor polynomial, Triple integral.