AN ANALYTICAL STUDY ON RADICAL THEORY IN RINGS - A REVIEW

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Abstract

Radical theory in rings is regarded as a major theoretical concept that specifies certain classical structure theorems for rings, enhancing profound understanding of the key aspects of ring theory. Radical theory has aided to the development of mathematics by providing a background to study and compare properties using radical-like theories, constructing rings with distinguished properties, and opening new dimensions for research. The research focused on gathering theories on radicals in rings purely from past research, filtering and combining them, that would help a reader by providing a brief yet clear understanding on the key concepts of radical theories. The research objectives are of four-fold: with respect to radicals in ring theory (i) to study general theories of radicals, (ii) to identify classical examples of radicals and their properties, (iii) to critically apply general theories of radicals to examples, (iv) and to find out the usefulness by generalizing the radical concept to abstract algebraic systems. The research method included collection of resources (electronic books and articles) from *online portals*, a methodical skimming of the resources, classification of the main topic into relevant subtopics, writing and compiling the grabbed knowledge into own appropriate words, analysis of the connection between examples with valid reasonings, and making of a final review presentation by assembling all the gathered information into appropriate components. In general, radical (P(R)) of a ring (R) is an ideal of "bad" elements of the ring (R) with an inappropriate or bad property (P). Inventive technique is used to classify bad elements in a ring and find radicals to make them good, reducing the radical of a quotient ring (R/P(R)) to zero ideal; P(R/P(R)) = (0). Key examples of radicals are Jacobson radical, Nilradical, Levitzki radical, Artinian radical, Brown-McCoy radical, Baer radical, etc. The bad property of these examples is generally related to zero multiplication or nilpotency. For a ring R, different kinds of radicals can be related by inclusions; $B(R) \subseteq L(R) \subseteq J(R) \subseteq G(R)$, where B denotes the Baer radical, L denotes the Levitzki radical, J denotes the Jacobson radical, and G denotes the Brown-McCoy radical. Theories were developed using the concepts of minimal right ideals, nilpotency and radical like-theories. With the modern research, mathematicians are generalizing the radical concepts to other algebraic systems like Banach algebra, Group algebra, Near rings, Lattices, etc.

Keywords: *bad property, radical, ring*