FEAR'S INFLUENCE ON PREDATOR-PREY INTERACTIONS

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ABSTRACT: *Interactions between animals and their surroundings are essential to the survival and success of individual organisms as well as entire ecosystems. The dynamic relationship between predators and their prey in an ecosystem is referred to as" exploitative interactions." The relationship between predators and prey is important in shaping the dynamics of an ecosystem and can have a significant impact on predator and prey populations. Predator-prey interactions can take many different forms and are influenced by a variety of factors, including the size and behavior of the predator and prey, the availability of resources, and the presence of other predators and prey species. The fear effect in predator-prey interaction refers to the idea that prey animals may exhibit altered behavior due to the perceived risk of predation. This can include changes in movement patterns, foraging behavior, and reproductive strategies. The fear effect is incorporated into mathematical models recently as it affects the rates of prey reproduction. In this work, we consider the classic two-species predator-prey model, where one of the species is "fearful" of the other when exactly the predation happens.*

Keywords: Predator-Prey, fear, survival radius

1. INTRODUCTION

Understanding the inherent nature and key behavior of many biological systems in our environment requires mathematical modeling. Researchers are drawn to mathematical modeling and analysis of exploitable biological resources. Predator-prey interactions are an important issue in ecology because they help to preserve ecological equilibrium. Researchers can forecast the consequences of changes in environmental conditions on population dynamics, connections between various species, and the overall ecosystem by using mathematical models. These models are useful for investigating biologically intriguing topics and comprehending complicated biological systems [1].

For more than a century, scholars have been interested in predator-prey interactions, and the Lotka-Volterra model is one of the first and most prominent mathematical models to represent these relationships [2]. Two pioneering mathematicians, Alfred Lotka and Vito Volterra, independently proposed the concept in the 1920s. The Lotka-Volterra model is the most basic and straightforward mathematical representation of predator-prey relationships. It consists of two first-order, non-linear differential equations, which are frequently used to comprehend the dynamics of biological systems in which one species acts as the predator and the other as the prey. The model's biggest benefit is its simplicity, which makes it simple to comprehend and use in a variety of biological systems. It provides a simple explanation for the population cycles of prey and predators, which exhibit periodic rises and reductions in densities. The model implies that in the absence of predators, the prey population increases exponentially, and the predator population is completely dependent on the availability of prey. This model has the following assumptions [3, 4].

- Prey has an infinite supply of food to eat. If there are no predators, the prey population increases at an exponential rate.

- Predators only consume certain prey, and that is their only source of sustenance. If there is no prey, the predator will die from starvation.
- Both populations' rates of change are proportionate to their sizes.
- A predator can eat an unlimited amount of the same type of prey.
- Both populations have no environmental problems. That is, no extraneous forces exist. Diseases, climate and weather fluctuations, pollution, and so forth are examples.

Under the above assumptions, Lotka Volterra basic predator-prey model is given by [3],

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad (1)$$
$$\frac{dy}{dt} = -\delta y + \gamma xy \qquad (2)$$

Parameter $\alpha, \beta, \delta, \gamma$ are positive constants. In particular, α and β are respectively prey increase rate in the absence of predator and mortality rate of those devoured by predators. Absence of prey, the predator dies at rate δ, and γ measures the skill of the predator in catching prey that turns into offspring. In general, the model exhibits stable behavior when the predator and prey populations oscillate around a stable equilibrium point. This equilibrium point is a stable limit cycle (a point at which the populations oscillate around a closed loop). On the other hand, the model can exhibit unstable behavior if the predator and prey populations grow without bounds.

According to evidence from numerous fields, the population dynamics are significantly impacted by not only direct killing but also the indirect effects of predator species on prey species. Any habitat's prey reacts to the perceived risk of predation by engaging in various anti-predator behaviors such as new habitat choice, foraging habits, vigilance, and several psychological changes [5]. In recent years, ecologists and mathematical biologists have become more interested in the non-consumptive effects of predation caused by fear of predators [6].

Risk-sensitive foraging, which refers to the assumption that prey animals may alter their foraging behavior in response to perceived levels of risk from predators is used in predator-prey models to reflect fear [7]. For example, prey animals may be more likely to forage in areas with good visibility or near protective cover if they feel that the risk of being attacked is high. For example, if the prey is more fearful and spends much more time undercover for protection, they could be less vulnerable to being preyed upon, which would reduce the number of predators. Because there are fewer predators around to attack the prey population, this can then result in less fear among them, and the cycle can continue. Much of the literature portrays the fear effect as a density-dependent variable that negatively affects prey populations [8, 9, 10, 11].

However, in this work, we looked at how the fear factor and its resulting emotional vulnerability is played out in the predator-prey interaction time creating more food for the predator and less survival for the prey. We will specifically look into how the dynamics of predator-prey systems are impacted by the emotional vulnerability, or how emotionally vulnerable a prey is to a predator while in an attack. To the best of our knowledge, no prior work has been done to study the fear effect on predator- prey interactions. Thus, in this work, we demonstrate the impact through the basic Lotka Volterra Model (Eqs. (1), (2)).

2. MODEL SIMULATION

In the works of [10, 11], the fear factor has quantified by $F(k, y) = \frac{1}{1+ky}$ which reduces the growth rate α of prey population, where $k, 0 \le k \le 1$, is the level of fear which drives anti-predator behavior of prey.

However, in our case, due to the fear of the prey, when the predators hunt the prey, the death rate of the prey devoured by the predators increases. Thus, the fear effect takes the form of $G(k, y) = (1 + ky)$ where $0\ k \leq 1$ is the level of fear of prey when it exposes to predator. The following conditions are satisfied by $G(k, y)$.

- $G(0, y) = 1$ (There is no fear of predation when a predator attacks prey $(k = 0)$. As a result, there is no additional increment of the death rate of prey (β) which are eaten by the predator.)
- $G(k, 0) = 1$ (The system has no predators $(y = 0)$. As a result, there is no increment in prey death rate (β) as a result of predator fear.)
- $\frac{\partial}{\partial k} G(k, y) > 0$ (When fear levels rise, prey death rate (β) due to predation also rises.)
- $\frac{\partial}{\partial y} G(k, y) > 0$ (Prey death rate (β) due to predation as the size of the predator population in- creases.)

With these conditions on the fear factor on interaction, the new model can be stated as below:

$$\frac{dx}{dt} = \alpha x - \beta(1 + ky)xy \tag{3}$$

$$\frac{dy}{dt} = -\delta y + \gamma xy \tag{4}$$

3. EQUILIBRIUM ANALYSIS

3.1. The Lotka Volterra model

As we mentioned in the introduction, the Lotka-Volterra predator-prey model [1, 2] depicts the interaction of two animal species living in the same habitat, and the number of each animal group is determined by two factors. Such as the birth or death rate or the number of successful encounters. Within-species rivalry has been disregarded in this case. And this approach only works when certain key assumptions are met [1].

To examine the behavior of this basic model, we must first solve the Lotka-Volterra model and assess its stability. We can begin by looking for steady-state solutions. This occurs when both derivatives are 0. When this occurs, the populations of the two species remain stable. These kinds of solutions are known as equilibrium points. Then we get equilibrium points as $E_0 = (0, 0)$ and $E_1 = (\frac{\delta}{\gamma}, \frac{\alpha}{\beta})$.

To investigate the stability of the system (Eqs. (1), (2)) at each equilibrium, the Jacobian matrix will be computed. By substituting the equilibrium points into the Jacobian matrix, we can find the corresponding eigenvalues. For 1st equilibrium point $E_0 = (0, 0)$, we get eigenvalues as $\lambda_1 = \alpha$ and $\lambda_2 = -\delta$. Since these are two distinct eigenvalues with opposite signs, here E_0 will be the saddle point, and it is always unstable. In this scenario, both species are endangered. We get 2nd equilibrium point as $E_1 = (\ \delta\ , \alpha\ )$. In this case, we gain eigenvalues as $\lambda_1 = +\sqrt{\alpha\delta}i$ and $\lambda_2 = -\sqrt{\alpha\delta}i$. Because α and β are positive according to our system (Eqs. (1), (2)), we get complex eigenvalues with a real part of zero. The trajectories in this situation neither converge to the critical point nor travel infinitely far away. Instead of that, they maintain steady, elliptical orbits. This is referred to as the center point (E_1). That implies, in this situation, the number of preys equals exactly the amount of food required to maintain the predator population steady.

3.2. Lotka Volterra model with fear in interaction

When the predator attacks the prey, we modify the fundamental Lotka Volterra model (Eqs. (1), (2)) to create a frightening impact. Then the new model will be developed as [4]. Here also, to examine the behavior of this model, we must first solve this modified model and assess its stability. To assess the behavior of this new model, we must find equilibrium points. They are given below:

$$\frac{dx}{dt} = 0 : x = 0 \; or \; y = \frac{-\beta+\sqrt{\beta^2+4\alpha\beta k}}{2\beta k} \; or \; y = \frac{-\beta-\sqrt{\beta^2+4\alpha\beta k}}{2\beta k} \tag{5}$$

$$\frac{dy}{dt} = 0 : x = \frac{\delta}{\gamma} \; or \; y = 0 \tag{6}$$

Then we get three equilibrium points as $E_0 = (0,0)$, $E_1 = (\frac{\delta}{\gamma}, \frac{-\beta+\sqrt{\beta^2+4\alpha\beta k}}{2\beta k})$ and $E_2 = (\frac{\delta}{\gamma}, \frac{-\beta-\sqrt{\beta^2+4\alpha\beta k}}{2\beta k})$. To investigate the stability at each equilibrium, the Jacobian matrix is computed. $J(E_i)$ is the Jacobian matrix of system (Eqs. (4)) at the equilibrium $E_i = (xi, yi)$ where $i = 0,1,2$.

$$J(E_i) = \begin{pmatrix} \alpha - \beta y_i(1 + k y_i) & -\beta x_i - 2\beta k x_i y_i \\ \gamma y_i & -\delta + \gamma x_i \end{pmatrix} \tag{7}$$

By substituting the equilibrium points to the $J(E_i)$ in Eq. (7), we can find the corresponding eigenvalues. E_0 gives the following Jacobian and eigenvalues:

$$J(E_0) = \begin{pmatrix} \alpha - \beta y_i(1 + k y_i) & -\beta x_i - 2\beta k x_i y_i \\ \gamma y_i & -\delta + \gamma x_i \end{pmatrix}; \; \lambda_1 = \alpha, \lambda_2 = -\delta \tag{8}$$

Since these are two distinct eigenvalues with opposite signs, E0 is a saddle point, and it is always unstable. In this scenario, both species are endangered.

$$J(E_i) = \begin{pmatrix} \alpha - \frac{-\beta+\sqrt{\beta^2+4\alpha\beta k}}{2k} - \frac{(-\beta+\sqrt{\beta^2+4\alpha\beta k})^2}{4\beta k} & -\frac{\beta\delta}{\gamma} - \frac{\delta}{\gamma}(-\beta + \sqrt{\beta^2 + 4\alpha\beta k}) \\ \frac{\gamma}{2\beta k}(-\beta + \sqrt{\beta^2 + 4\alpha\beta k}) & 0 \end{pmatrix};$$

$$\lambda_1 = -\frac{\sqrt{2}\sqrt{\frac{\delta\left(4\alpha k+\beta-\sqrt{\beta^2+4\alpha\beta k}\right)}{k}}}{2} i, \; \lambda_2 = \frac{\sqrt{2}\sqrt{\frac{\delta\left(4\alpha k+\beta-\sqrt{\beta^2+4\alpha\beta k}\right)}{k}}}{2} i \tag{9}$$

All eigenvalues are complex without real part if the following condition is satisfied,

$$4\alpha k + \beta \geq \sqrt{\beta(4\alpha k + \beta)} \tag{10}$$

The trajectories do not converge to the critical point in this scenario, nor do they go infinitely. In this case, both population dynamics achieve a neutrally stable orbit. The third equilibrium point is not analyzed since it is infeasible (y < 0) for the context of our problem.

4. NUMERICAL RESULTS AND DISCUSSION

We first showcase the results of the basic LV model for comparisons with the fear factor afterward.

4.1. The Lotka Volterra model

The following example initial conditions and parameter values are used to demonstrate the numerical solution of the basic Lotka Volterra predator-prey model (Eqs. (1), (2)). The parameter values and ICs taken are respectively $\alpha = 1.2, \beta = 0.6, \delta = 0.3$ and $x_0 = 2$ and$y_0 = 1$. The differential equation system is then solved with Runga Kutta 4th order method. According to dynamics given in Fig.4.1, Fig.4.2, this model has a periodic pattern. Naturally, an increase in prey leads to an increase in predators. However, as predators increase, prey populations decline and reach a minimum, causing predators to decline. As predators decrease, new prey develops [12]. This dynamic creates a continuous cycle of growth and decline. The phase diagram (Figure4.2) also clearly depicts the periodic pattern. The closed curve demonstrates the two species' periodic relationship.

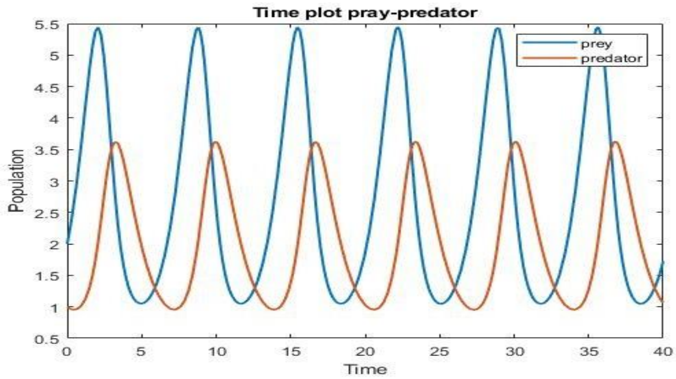

Figure 4.1: Prey and Predator abundance over time simulated by basic LV model. The parameter values and initial conditions taken are respectively $\alpha = 1.2, \beta = 0.6, \delta = 0.3$ and $X_0 = 2$ and $Y_0 = 1$

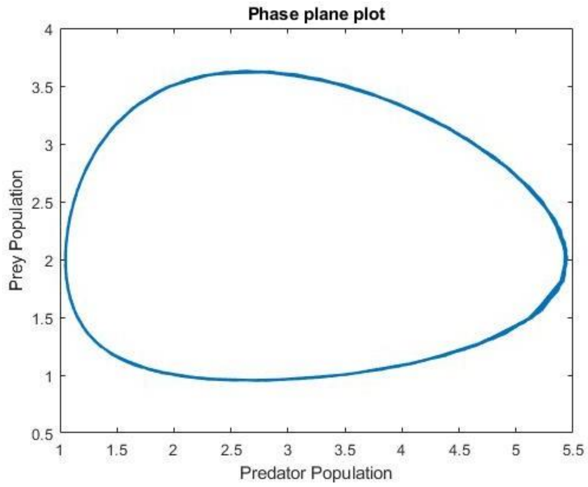

Figure 4.2: Phase diagram corresponding to model simulations in Fig. 4.1

Each population may be zero only if the other population is likewise zero. Species cannot become extinct at various periods; they must all die extinct at the same time. However, the closed shape of the phase plot indicates that this condition is never encountered in this model.

4.2. Lotka Volterra model with fear in interaction

The numerical solution of the Lotka Volterra model with fear in interaction is demonstrated using the following example of initial conditions and parameter values. Here also we took parameter values as $\alpha = 1.2, \beta = 0.6, \delta = 0.8, \gamma = 0.3$ and $x_0 = 2$ and $y_0 = 1$. We picked several values for the fear parameter in its full range with 0.2 equal intervals.

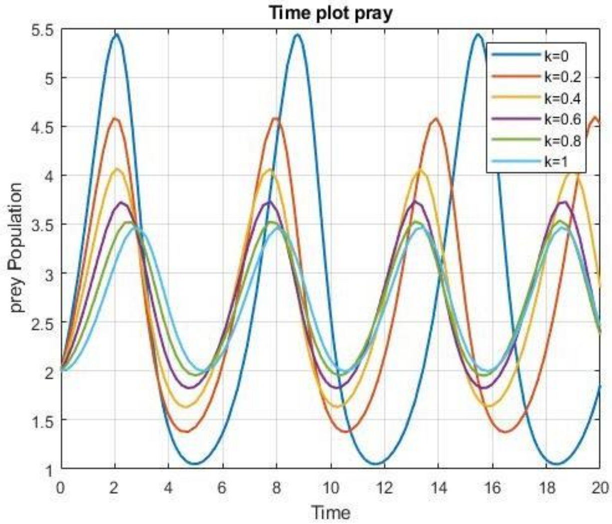

Figure 4.3: Prey abundance over time when the fear at different levels is present in the predator-prey interactions. The parameters and initial conditions were $\alpha = 1.2, \beta = 0.6, \delta = 0.8, \gamma = 0.3$ and $x_0 = 2$ and $y_0 = 1$.

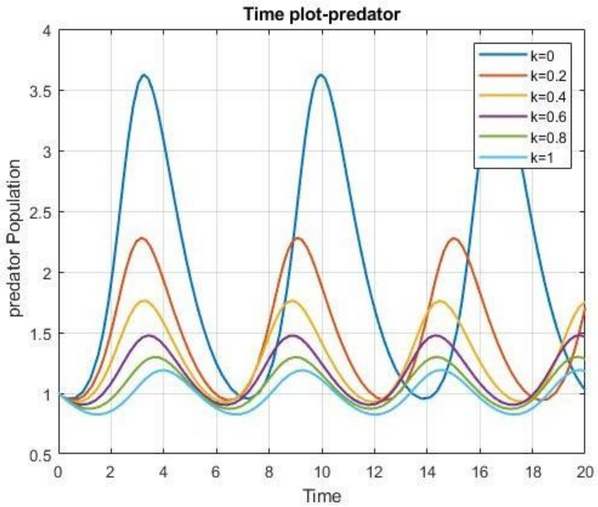

Figure 4.4: Predator abundance over time when the fear at different levels is present in the predator-prey interactions. The parameters and initial conditions were $\alpha = 1.2, \beta = 0.6, \delta = 0.8, \gamma = 0.3$ and $x_0 = 2$ and $y_0 = 1$.

According to the stability conditions of the analytical solution (Eq. 10), $4\alpha k + \beta \geq \sqrt{\beta(4\alpha k + \beta)} = 1.56 > 0.967$ when $k = 0.2$. As we can see from the Figures, Fig. 4.3 and Fig. 4.4, when the fear effect increases, the variance of both prey and predator population decreases. But the periodic pattern of population abundance does not change. As k increases, the frequency of the cyclic pattern also increases. Because when both populations are low, interactions between predator and prey occur quickly.

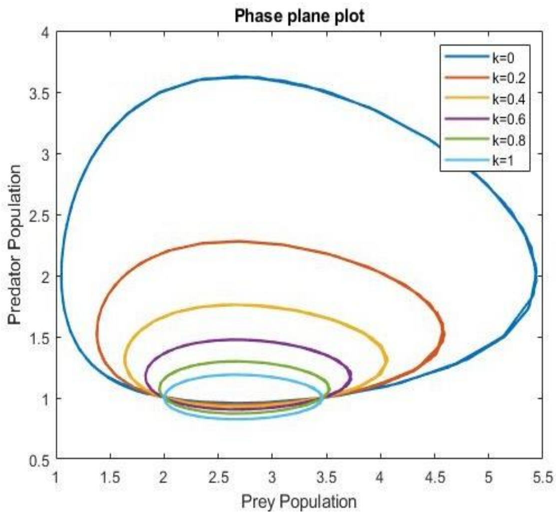

Figure 4.5: Phase diagram (Other parameters remain unchanged)

According to the phase portrait in Fig.4.5 of the new model, as the level of fear increases, the animal population's survival radius gradually converges into small population groups. Also, phase portraits at each k value vividly highlight the periodic pattern of two species.

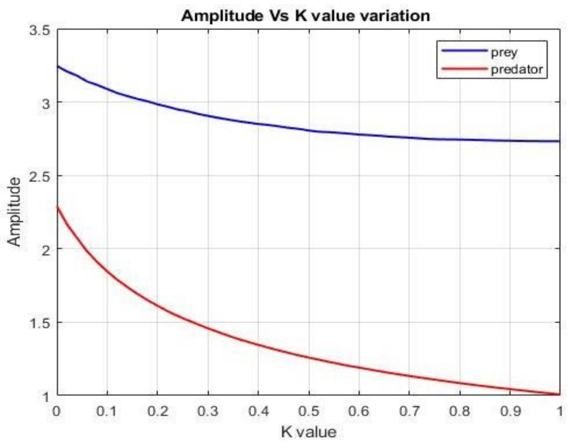

Figure 4.6: Variation of the amplitude of predator and prey abundance with k

The amplitude of the periodic pattern of Figure 4.3 and Figure 4.4 gradually decreases and becomes constant after some value of k. This idea emphasizes that as the level of fear increases, the strength of the predator-prey interaction weakens (Figure 4.6). This is because when prey animals become more fearful, they may exhibit behaviors that reduce their chances of capture or consumption, such as greater vigilance, faster running speeds, or frequent hiding.

5. CONCLUSION

Recent field research shows that fear induced by predators affects the intrinsic growth rate of prey species in an ecosystem [5]. Also, the intensity of fear fluctuates depending on various conditions such as mating behavior, food availability, internal competition, etc. In here first, we looked at the fundamental Lotka Volterra model [1,2]. The model features periodic patterns depending on its equilibrium point and stability criteria, and these dynamics form a cycle of continual growth and decrease (Figure 4.2). We suggested and examined the complex dynamical behavior of a predator-prey model in this study, including the influence of fear when a predator attacks the prey [4]. According to our findings, when the level of fear felt by prey when predation happens increases, the variance of both prey and predator abundance will decrease. The cyclic pattern's frequency rises as well (Figure4.3, Figure4.4). When the k value is greater than 0.7, the amplitude of the periodic pattern progressively declines and becomes constant (Figure 4.6). Numerical simulations validate our analytical conclusions. Further research is needed to overcome the limitations of this study and expand our knowledge of the function of fear in predator-prey interactions. Future research should employ more ecologically relevant stimuli to investigate the impact of fear on predator and prey behavior in real settings. Future studies should also look at the potential trade-offs between fear-induced behavioral changes and other fitness-related variables like foraging efficiency or reproductive success.

REFERENCES

[1] Abrams, P. A. (2000). The evolution of predator-prey interactions: Theory and evidence. *Annual Review of Ecology and Systematics, 31*(1), 79-105. doi: 10.1146/annurev.ecolsys.31.1.79

[2] Lotka, A. J. (1920). Analytical note on certain rhythmic relations in Organic Systems. *Proceedings of the National Academy of Sciences, 6*(7), 410-415. doi:10.1073/pnas.6.7.410

[3] LAHAM, M. F., KRISHNARAJAH, I., & JUMAAT, A. K. (2012). A numerical study on Predator Prey Model. *International Journal of Modern Physics: Conference Series, 09*, 347-353. doi:10.1142/s2010194512005417

[4] Wangersky, P. J. (1978). Lotka-Volterra population models. *Annual Review of Ecology and Systematics, 9*(1), 189-218. doi:10.1146/annurev.es.09.110178.001201

[5] Roy, J., & Alam, S. (2020). Fear factor in a prey–predator system in deterministic and stochastic environment. *Physica A: Statistical Mechanics and Its Applications, 541*, 123359. doi:10.1016/j.physa.2019.123359

[6] Stankowich, T., & Blumstein, D. T. (2005). Fear in animals: A meta-analysis and review of Risk Assessment. *Proceedings of the Royal Society B: Biological Sciences, 272*(1581), 2627-2634. doi:10.1098/rspb.2005.3251

[7] Srivastava, V., Takyi, E. M., & Parshad, R. D. (2023). The effect of "fear" on two species competition. *Mathematical Biosciences and Engineering, 20*(5), 8814-8855. doi:10.3934/mbe.2023388

[8] Wang, X., Zanette, L., & Zou, X. (2016). Modelling the fear effect in predator–prey interactions. *Journal of Mathematical Biology, 73*(5), 1179-1204. doi:10.1007/s00285-016-0989-1

[9] Antwi-Fordjour, K., Parshad, R. D., Thompson, H. E., & Westaway, S. B. (2023). Fear-driven extinction and (de)stabilization in a predator-prey model incorporating prey herd behavior and mutual interference. *AIMS Mathematics, 8*(2), 3353-3377. doi:10.3934/math.2023173

[10] Wang, X., Tan, Y., Cai, Y., & Wang, W. (2020). Impact of the fear effect on the stability and bifurcation of a Leslie–Gower Predator–Prey Model. *International Journal of Bifurcation and Chaos, 30*(14), 2050210. doi:10.1142/s0218127420502107

[11] He, M., & Li, Z. (2022). Stability of a fear effect predator–prey model with mutual interference or Group Defense. *Journal of Biological Dynamics, 16*(1), 480-498. doi:10.1080/17513758.2022.2091800

[12] C., A. (2021, October 31). Prey and predators‐a model for the dynamics of Biological Systems. Retrieved December 20, 2022, from https://towardsdatascience.com/prey-and-predators-a-model-for-the-dynamics-of-biological-systems-747b82d2ea9e