INVESTIGATING THE FORECASTING PERFORMANCE OF GARCH MODELS IN PREDICTING FINANCIAL VOLATILITY OF LARGE-CAP STOCK INDICES DURING THE COVID-19 PANDEMIC

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ABSTRACT: - *Financial volatility forecasting is especially important in financial econometrics that helps Investors to minimize their losses by understanding the future financial volatility. To predict financial volatility GARCH models are used which better reflects the leverage effect, volatility clusters and volatility jumps in a financial time series. COVID-19 pandemic is one of the most extreme events we have faced recently which led to sudden fall in stock prices and financial instability. While many research papers have focused on stock indices particular to relevant countries or emerging markets, this study examines large-cap stock indices, which are safer to invest in and covering almost the entire world by considering ten major large-cap stock indices from nine countries which represent seven regions; Asia, South Asia, Europe, Middle East, North America, Oceania and South America. Statistical loss functions; MSE, RMSE, MAE, R^2 Log, and QLIKE, were used to determine the best model out of GARCH, GJR-GARCH, EGARCH, and PGARCH. Although the literature review suggests using the High-Low proxy method to capture realized financial volatility, this study used the OHLC(Open-High-Low-Close) volatility estimator, which considers drift-independence and is capable of handling opening-pricing-jumps. All tests were conducted using python programming for significance levels of 1%, 5%, and 10%, and the results indicated that the study is statistically significant at the 1% level. The results reveal that, although the stock indices represent different regions, they have shown a similar impact towards COVID-19, while NZ50 and Nikkei225 indices slightly differ as New Zealand and Japan are less affected by COVID-19 during the period 11.03.2020 – 10.03.2022. The study concludes that, averagely the best model to forecast financial volatility on large-cap stock indices which affected from COVID-19 is EGARCH (1,1) as it is asymmetric and able to grasp the leverage effect in a crisis. GJR-GARCH (1,1) is preferred by Nikkei225 and NZ50.*

Keywords: - GARCH, financial volatility, large-cap stock indices, forecasting, COVID-19

1. INTRODUCTION

Financial volatility forecasting is a critical area of financial econometrics that has received considerable attention in recent years. Investors can minimize their losses by understanding the future financial volatility. GARCH models have arisen as the most popular tool used in mathematical finance to forecast financial volatility, as they better reflect the leverage effect, volatility clusters and volatility jumps in a financial time series as well as volatility varies within a specific fixed range (Tsay, 2014). The main objective of this research project is to evaluate which GARCH model achieves the best predicted financial volatility closest to the realized financial volatility on large-cap stock indices during a crisis like COVID-19. Variations of the GARCH model; GARCH, GJR-GARCH, EGARCH and PGARCH are used to develop and forecast the financial volatility using python programming. GARCH model has more flexible lag structure, it uses fewer parameters and good for volatility clustering and leptokurtosis. But since it is symmetrical, it fails to capture the effect of leverage (Belisle, 1986). EGARCH and GJR-GARCH models are asymmetric, and they can capture the leverage effect as negative shock in asset return of the financial series' volatility having a larger effect. EGARCH imposed the natural logarithm of conditional variance. GJR-GARCH implement in practice since the variance is directly modelled instead of using the natural logarithm. PGARCH enhances the goodness of fit of the model, which can be argued as a reasonable approach when forecasting (Ding, 2011).

Previous studies have mainly focused on particular stock indices within relevant countries (Angabini & Wasiuzzaman, 2011) or emerging markets (Srinivasan & Ibrahim, 2010). In contrast, this study examines ten major large-cap stock indices from nine countries, representing seven regions; Asia, South Asia, Europe, Middle East, North America, Oceania and South America, that covers almost whole world to review whether there is a different impact on indices as they are representing different countries. Also, the study observes the financial volatility behaviour of two stock indices from the same country (US), three stock indices from same region (European).

The models examined in this study are GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and PGARCH(1,1) on large-cap stock indices; CAC40 (France), DAX (Germany), Dow30 (USA), FTSE100 (UK), IBOVESPA (Brazil), Nikkei225 (Japan), NZ50 (New Zealand), S&P500 (USA), S&P BSE SENSEX (India) and TA125 (Israel). Forecasting performance of the models is analyzed by using statistical loss functions; Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Quasi-Likelihood (QLIKE) Loss function, R^2 Log and Mean Absolute Error (MAE). The result of the research will evaluate the best optimal GARCH model for predicting the financial volatility in such a crisis.

2. METHODOLOGY

Market capitalization with over USD 10 billion has been considered as large-cap stocks. For this study, daily adj. close, high, low, open and close prices of selected large-cap stock indices are collected from Yahoo Finance for the time period from 11th March 2015 to 10th March 2022.

As the World Health Organization declared the outbreak of COVID-19, a pandemic, on 11th March 2020, data from 11th March 2015 to 10th March 2020 is used as in-sample data (before the pandemic) to estimate and forecast the financial volatility in the out-of-sample (during the pandemic) and is compared with the out-of-sample data from 11th March 2020 to 10th March 2022. Daily log returns (R_t) are used as the variable for this study and P_t and P_{t-1} are the current day (t) and previous day (t-1) adj. closing prices.

$$R_t = \log(P_t) - \log(P_{t-1}) \quad (1)$$

Figure 1 : Daily adj. closing prices of the stock indices

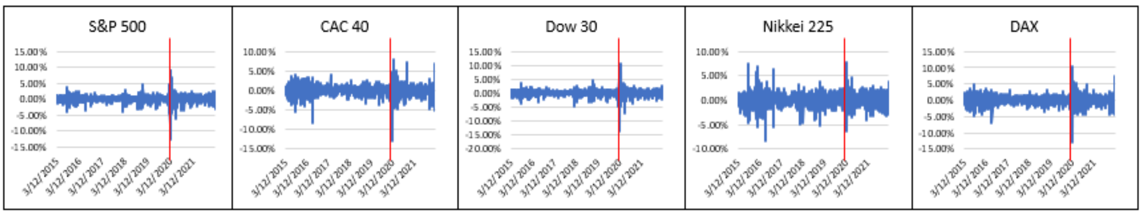

Figure 2 : Daily log-returns of the stock indices

The red colored vertical line represents the date 11th March 2020, which was when COVID-19 caused a downward movement in the prices of the indices. The observations before the red-colored lines are the in-sample and the observations after are the out-of-sample. The Figure 1 depicts that the selected stock indices seem to be more affected by COVID-19, causing a large price drop and Figure 2 depicts that there is a big increase in variance that converges back towards the normal level during the crisis.

All models have been specified with the smallest lag order since this provides adequate results and makes the models more comparable. Significance levels of 1%, 5% and 10% for this study are considered as they are the most common and as many research papers have suggested. Python programming with the appropriate packages and the MS Excel are used for the calculation of the study. For the convenience, only five stock indices; S&P500, CAC40, Dow30, Nikkei225 and Dax are included for secondary results of the research.

1.1 Quantitative Approach

a. *Data Analysis and Model Fitting for the In-sample Data*

First, to analyze and model a data in a time series, mean and variance should not vary across time. Hence, stationarity of the financial time series of the stock indices is examined by conducting Augmented Dickey Fuller (ADF) test in python for the significance levels of 1%, 5% and 10%. The goodness of fit of the models is examined by conducting Ljung-Box Test using "acorr_ljungbox" syntax in python at the levels of 1%, 5% and 10% significance. This tests the lack of fit of time-series models by examining if there is sequential correlation on the residuals of the ARMA (p, q) model as the values of the lags of return should be dependent, but sequentially uncorrelated. The relevant order of p and q in the conditional mean model ARMA was determined using "auto.arima" syntax in python. Then the goodness-of-fit of the ARIMA model is evaluated using the Akaike Information Criterion (AIC) as it reflects the approximation of the reality of the situation. The model with the lowest AIC value is the best.

$$AIC = -2\log(L) + 2(p + q + d) \qquad (2)$$

L is the likelihood of the data series, p is the order of the autoregressive part, q is the order of the moving average part and d is the intercept of the ARIMA (p, d, q) model. To find the goodness-of-fit of the ARIMA model, Bayesian Information Criterion (BIC) can also be used. As BIC attempts to find the perfect fit, it is not realistic to real-life data, and as most researchers suggested AIC is better for large samples.

Then conducted ARCH-LM test using "stats.diagnostic.het_arch" syntax in python to determine the presence of ARCH effects in the residuals of the in-sample data of the stock indices, as if ARCH effects are detected, the GARCH models are the appropriate framework for the financial time-series data. If not, answers will vary and GARCH models will not be good to use to model the time series data. Since the financial time series of the stock indices have fat tails, positive returns and negatively skewed, Student's t-distribution is used to fit the error term ε_t (Kumari and Tan, 2013). The models, GARCH, EGARCH, GJR-GARCH and PGARCH are fitted to the financial time series of all the stocks using "model.fit" syntax with Standardized Student's t-distribution in python.

b. *Model Forecast for the Out-of-sample Data*

One-step-ahead Robust Recursive forecasting approach is conducted in python to forecast financial volatility in out-of-sample as only one model is required and saves computational time. The window is set for no.of trading days during the period 11.03.2020 to 10.03.2022 in each stock indices. Hence, the window = 504 for S&P500, Dow30 and FTSE100, window =

494 for S&P BSE SENSEX and IBOVESPA, window = 487 for Nikkei225 and TA125, window = 514, 508 and 500 for CAC40, DAX and NZ50.

c. Estimation of Realized Financial Volatility

To compare the predicted financial volatility by the GARCH models, a realized financial volatility proxy was calculated using the developed model of (Open-High-Low-Close) OHLC volatility estimator by Yang D. and Zhang Q. in 2000 which is drift-independent and able to handle opening pricing jumps (Yang and Zhang, 2000).

$$\sigma^2_{Yang\ Zhang} = \sigma^2_{Open} + k\sigma^2_{Close} + (1-k)\sigma^2_{RS}, \text{ where } k = \frac{0.34}{1.34+\frac{T+1}{T-1}} \quad (3)$$

Overnight volatility $$\sigma^2_{Open} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\ln\left(\frac{O_t}{C_{t-1}}\right) - \overline{\ln\left(\frac{O_t}{C_{t-1}}\right)}\right)^2 \quad (4)$$

Open-to-Close volatility $$\sigma^2_{Close} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\ln\left(\frac{C_t}{O_t}\right) - \overline{\ln\left(\frac{C_t}{O_t}\right)}\right)^2 \quad (5)$$

The volatility estimator proposed by Rogers and Satchell in 1991 and its developed and simplified volatility estimator,

$$\sigma^2_{RS} = \frac{1}{T}\ln\left(\frac{H_t}{O_t}\right)\ln\left(\frac{H_t}{C_t}\right) + \ln\left(\frac{L_t}{O_t}\right)\ln\left(\frac{L_t}{C_t}\right) \quad (6)$$

$H_t, L_t, O_t\ and\ C_t$ stands for the highest price, lowest price, opening price and the closing price of the current trading day t and T is the number of days in the financial time series. Therefore, for this study, OHLC volatility estimator developed by Yang and Zhang is calculated using MS Excel and T is set for the number of trading days of the stock indices for the period 11th March 2020 to 10th March 2022 in each stock indices. Hence, the window = 504 for S&P500, Dow30 and FTSE100, window = 494 for S&P BSE SENSEX and IBOVESPA, window = 487 for Nikkei225 and TA125, window = 514, 508 and 500 for CAC40, DAX and NZ50 respectively.

d. Forecast Evaluation using Statistical Loss Functions

A loss function summarizes the forecast errors, difference between the realized and the forecasted financial volatility, which provides a measurement of how well the prediction matches the observed data. To select the optimal forecast model, five statistical loss functions; MSE, RMSE, MAE, QLIKE and R^2 Log are used for the study as different loss functions penalize differently. The model with the smallest loss value indicates the model that has the closest predicted financial volatility to the realized financial volatility.

3. RESULTS AND DISCUSSION

Table 1 : Descriptive statistics for in-sample data

Stock Index	Country	Region	Observations	Mean	Std	Min	Max	Skewness	Kurtosis
S&P 500	USA	North America	1258	0.000275	0.00924	(0.07901)	0.0484	(0.8516)	8.4099
CAC 40	France	Europe	1277	(0.000059)	0.01098	(0.08764)	0.0406	(1.0075)	7.4200
Dow 30	USA	North America	1258	0.000278	0.00936	(0.08106)	0.0497	(0.8482)	8.8141
Nikkei 225	Japan	Asia	1221	0.000049	0.01246	(0.08253)	0.0743	(0.3908)	5.9617
Dax	Germany	Europe	1262	(0.000095)	0.01141	(0.08277)	0.0485	(0.7331)	4.4173
FTSE 100	UK	Europe	1264	(0.000095)	0.00917	(0.07999)	0.0352	(0.8242)	6.6126
S&P BSE SENSEX	India	South Asia	1226	0.000178	0.00869	(0.06120)	0.0519	(0.4340)	4.7111
NZ 50	New Zealand	Oceania	1254	0.000494	0.00607	(0.03710)	0.0241	(0.7586)	3.4033
IBOVESPA	Brazil	South America	1235	0.000514	0.01469	(0.12981)	0.0690	(0.7608)	7.2705
TA125	Israel	Middle East	1057	(0.000039)	0.00878	(0.08662)	0.0348	(2.1101)	14.7606

Table 1 summarizes that no.of observations is different as the no.of trading dates for each countries is different. The mean for CAC40, DAX, FTSE100 and TA125 is negative due to more price drops than price ups. There is a substantial difference between the minimum and maximum values. IBOVESPA has the largest difference. All samples have a negative skew and a higher kurtosis.

Table 2 : Results of the ADF Test for the stock indices

	S&P 500	CAC 40	Dow 30	Nikkei 225	DAX
1%	-3.43736	-3.43723	-3.43736	-3.43752	-3.43735
5%	-2.86463	-2.86458	-2.86463	-2.86470	-2.86463
10%	-2.56842	-2.56839	-2.56842	-2.56845	-2.56842
ADF Statistic	-35.20933	-16.74618	-35.60034	-14.01622	-36.12774

Results shown in the Table2 indicate that, since value of the Test Statistic is less than the critical value, H0 can be rejected at 1%, 5% and 10% of significance levels for stock indices stating that the financial time series of stock indices is stationary.

Table 3 : AIC values to determine suitable orders of p and q for ARMA Model

ARIMA (p, d, q)	Mean	AIC Value				
		S&P 500	CAC 40	Dow 30	Nikkei 225	DAX
(2,0,2)	non-zero	-6361.903	-5921.492	-6348.248	-5360.043	-5764.920
(0,0,0)	non-zero	-6362.422	-5924.026	**-6350.369**	-5365.380	-5769.741
(1,0,0)	non-zero	-6360.749	-5922.281	-6348.461	-5366.043	-5767.741
(0,0,1)	non-zero	-6360.787	-5922.293	-6348.460	-5366.037	-5767.741
(0,0,0)	zero	**-6362.952**	**-5925.983**	-6350.199	**-5366.984**	**-5771.713**
(1,0,1)	non-zero	-6359.178	-5920.287	-6346.358	-5364.058	-5765.741

The yellow-colored values represent the lowest AIC values and the red colored values represent the second lowest AIC values. According to the results from the Table 3, the best Model for S&P500, CAC40, Nikkei225 and DAX is, ARIMA(0,0,0) with zero mean and the best Model for Dow 30 is ARIMA(0,0,0) with non-zero mean. Therefore, Ljung Box Test is conducted for the residuals of the ARMA(0,0) with non-zero mean for Dow30 and ARMA(0,0) with zero mean for S&P500, CAC40, Nikkei225 and DAX, to examine the goodness of fit of the models.

Table 4 : Results of the Ljung Box Test for stock indices

Stock Index	S&P 500	CAC 40	Dow 30	Nikkei 225	DAX
p-value	0.2102	0.0284	0.1665	0.8033	0.0315
Test Statistic	9.6357	15.6649	10.4101	3.7928	15.3741

P-values highlighted in yellow color are less than the confidence interval at significance level of 5% and 10% and greater than the confidence interval at 1% significance level. Red colored p-values are greater than the confidence interval at the significance levels of 1%, 5% and 10%. According to the Table 4, the Ljung-Box test statistic is not significant for S&P500, Dow30 and Nikkei225 at significance levels of 1%, 5% and 10%. Hence, H0 can be rejected. Therefore, ARMA residuals exhibit no serial correlation for the indexes, S&P500, Dow30 and Nikkei225 at significance levels of 1%, 5% and 10%. Ljung-Box test statistic is significant for CAC40 and DAX indices at significance levels of 5% and 10%. But not at 1%. Hence, for all the stock indices, H0 can be rejected at 1% significance level as the p values are less than the confidence interval. Therefore, the lagged values of the ARMA(0,0) model are dependent, but serially uncorrelated at 1% significance level. As the models are fitted for the significance level of 1%, the study continued considering the 1% level of significance.

According to the results of ARCH-LM Test, H0 can be rejected for in-sample data as there are ARCH(q) disturbances in the residuals after the lags 497, 188, 498, 389 and 326 of S&P500, CAC40, DOW30, Nikkei225 and DAX respectively at 1% significance level. Therefore, the results conclude that ARCH effect is present for financial time series of indices at 1% significance level. Hence, GARCH models are appropriate to forecast the volatility of the financial time series of stock indices.

Table 5 : Coefficients of the models fitted for in-sample data of the selected 5 stock indices at the significance level of 1%

S&P 500					
Model	Omega	Alpha	Beta	Gamma	Delta
GARCH (1,1)	0.0239	0.2159	0.783		
EGARCH (1,1)	-0.0278	0.2162	0.9479	-0.2363	
GJR-GARCH (1,1)	0.0301	6.18E-17	0.7854	0.3807	
PGARCH (1,1)	0.0413	0.1491	0.8509	0.9997	0.8389

CAC 40					
Model	Omega	Alpha	Beta	Gamma	Delta
GARCH (1,1)	0.032	0.17	0.8257		
EGARCH (1,1)	-0.0063	0.122	0.9595	-0.2254	
GJR-GARCH (1,1)	0.0354	0	0.8326	0.3007	
PGARCH (1,1)	0.0401	0.1194	0.8771	0.9997	0.8855

Dow 30					
Model	Omega	Alpha	Beta	Gamma	Delta
GARCH (1,1)	0.0232	0.1994	0.797		
EGARCH (1,1)	-0.0194	0.2227	0.954	-0.1858	
GJR-GARCH (1,1)	0.0277	2.01E-03	0.8159	0.3029	
PGARCH (1,1)	0.0361	0.1239	0.8564	0.977	1.0745

Nikkei 225					
Model	**Omega**	**Alpha**	**Beta**	**Gamma**	**Delta**
GARCH (1,1)	0.1024	0.1875	0.7723		
EGARCH (1,1)	0.0158	0.1848	0.9172	-0.2481	
GJR-GARCH (1,1)	0.0923	2.3E-11	0.7727	0.3587	
PGARCH (1,1)	0.0795	0.1378	0.8367	0.9997	0.8299

DAX					
Model	**Omega**	**Alpha**	**Beta**	**Gamma**	**Delta**
GARCH (1,1)	0.0214	0.1061	0.8875		
EGARCH (1,1)	0.0021	0.1023	0.9644	-0.1856	
GJR-GARCH (1,1)	0.0277	2.8E-10	0.8836	0.199	
PGARCH (1,1)	0.0337	0.0878	0.9036	0.9997	0.9741

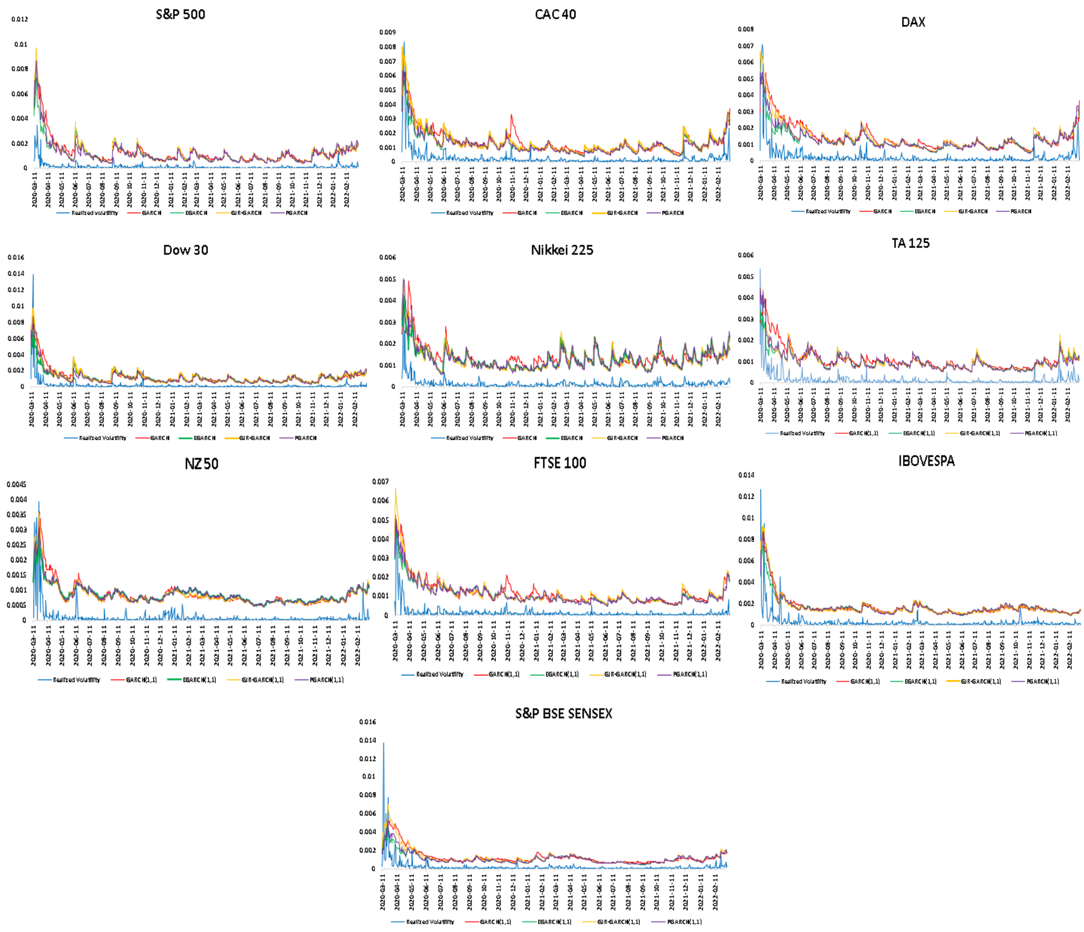

Figure 3 : Performance of the GARCH models on forecasted financial volatility compared to the realized financial volatility for each stock index during the period COVID-19

Blue colored line of the graphs represents realized financial volatility which is calculated using OHLC volatility proxy developed by Yang and Zhang. Red-, green-, yellow- and purple-colored lines represents the predicted financial volatility forecasted by the models GARCH, EGARCH, GJR_GARCH and PGARCH respectively which is calculated using one-step ahead Robust recursive window forecast approach.

According to the results, all models tend to less predictable the realized financial volatility. All predictions follow the movement of the realized financial volatility. GARCH (1,1) and GJR-

GARCH (1,1) are more volatile in their predictions during the more volatile period. EGARCH (1,1) and PGARCH (1,1) predictions varies closely together. GARCH (1,1) model has reacted a bit delayed to the fluctuations. Though the indices are from 9 different countries and 7 different regions, the COVID-19 crisis has been affected to all the ten stock indices almost equally as they are performing a large-cap, while NZ50 and Nikkei225 indices differ slightly as New Zealand and Japan are less affected by COVID-19 during the period 11.03.2020 – 10.03.2022. When considering the behaviour of the financial volatility forecasting performance, both S&P500 and Dow30 from the United States, show close movement. Germany's DAX, France's CAC40 and UK's FTSE100 indices also show almost same trend as they represent Europe and have a similar economy.

Table 6 : Evaluation of financial volatility forecasting performance of the GARCH models

S&P 500					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.711E-06	0.0013081	0.0010834	1024.99	9.47978
EGARCH (1,1)	1.229E-06	0.0011086	0.0009858	1002.89	9.18967
GJR-GARCH (1,1)	1.632E-06	0.0012774	0.0010442	1000.63	9.13050
PGARCH (1,1)	1.417E-06	0.0011905	0.0009882	984.90	8.93924

CAC 40					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.671E-06	0.0012928	0.0011533	895.86	7.87060
EGARCH (1,1)	1.182E-06	0.0010871	0.0009976	845.28	7.26714
GJR-GARCH (1,1)	1.583E-06	0.0012582	0.0011065	872.27	7.56108
PGARCH (1,1)	1.260E-06	0.0011224	0.0010057	840.48	7.19044

Dow 30					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.700E-06	0.0013040	0.0010834	1033.84	9.53005
EGARCH (1,1)	1.285E-06	0.0011336	0.0009900	1012.86	9.25519
GJR-GARCH (1,1)	1.574E-06	0.0012545	0.0010410	1009.87	9.18823
PGARCH (1,1)	1.402E-06	0.0011843	0.0009918	994.80	9.00793

Nikkei 225					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.681E-06	0.00129655	0.001216560	919.31	8.17059
EGARCH (1,1)	1.455E-06	0.00120605	0.001148346	897.30	7.92562
GJR-GARCH (1,1)	1.513E-06	0.00122993	0.001145550	887.00	7.78092
PGARCH (1,1)	1.549E-06	0.00124464	0.001165013	896.47	7.90802

DAX					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.758E-06	0.0013260	0.0011997	889.08	7.85134
EGARCH (1,1)	1.233E-06	0.0011102	0.0010422	848.34	7.38763
GJR-GARCH (1,1)	1.611E-06	0.0012693	0.0011488	871.92	7.63004
PGARCH (1,1)	1.319E-06	0.0011484	0.0010572	845.86	7.33029

TA125					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.073E-06	0.0010361	0.0009612	974.72	8.96025
EGARCH (1,1)	8.094E-07	0.0008997	0.0008573	941.02	8.55608
GJR-GARCH (1,1)	9.013E-07	0.0009494	0.0008813	940.33	8.52604
PGARCH (1,1)	8.576E-07	0.0009261	0.0008659	936.43	8.48539

IBOVESPA					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	2.607E-06	0.0016145	0.0014233	919.84	8.14266
EGARCH (1,1)	2.292E-06	0.0015139	0.0014002	923.22	8.17210
GJR-GARCH (1,1)	2.587E-06	0.0016083	0.0014160	918.57	8.11479
PGARCH (1,1)	2.536E-06	0.0015926	0.0014245	922.78	8.15763

NZ 50					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	6.887E-07	0.0008299	0.0007657	1381.40	13.30799
EGARCH (1,1)	6.416E-07	0.0008010	0.0007686	1394.84	13.49011
GJR-GARCH (1,1)	6.115E-07	0.0007820	0.0007325	1370.06	13.13875
PGARCH (1,1)	6.483E-07	0.0008051	0.0007653	1390.73	13.42440

FTSE 100					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.396E-06	0.00118154	0.001052595	1164.73	10.87891
EGARCH (1,1)	1.095E-06	0.00104624	0.000958770	1135.16	10.44381
GJR-GARCH (1,1)	1.367E-06	0.00116904	0.001023595	1147.51	10.63561
PGARCH (1,1)	1.140E-06	0.00106759	0.000959018	1128.37	10.35809

S&P BSE SENSEX					
Model	MSE	RMSE	MAE	QLIKE	R^2 Log
GARCH (1,1)	1.681E-06	0.0012966	0.0010773	827.09	6.98507
EGARCH (1,1)	1.204E-06	0.0010971	0.0009200	777.68	6.42769
GJR-GARCH (1,1)	1.641E-06	0.0012810	0.0010472	804.58	6.71670
PGARCH (1,1)	1.273E-06	0.0011284	0.0009325	776.72	6.42032

The values highlighted in yellow color represent the lowest value of each statistical loss functions which indicates the model that has the closest predicted financial volatility to the realized financial volatility and the values highlighted in red color represent the highest loss values which are not preferred to predict the financial volatility. The best model to predict each stock index according to the result of the statistical loss values are highlighted in green color.

As the results of the Table 5, the difference between the best and worst fitted model is generally small. The EGARCH model is preferred by the MSE, RMSE and MAE statistical loss functions for many stock indices. As indicated by the two statistical loss functions MSE and RMSE, the EGARCH is closer in its prediction of outliers than the other models as MSE and RMSE are more sensitive to outliers. PGARCH is the model preferred by QLIKE and R^2 Log. EGARCH, PGARCH and GJR-GARCH are the best models on average predict the realized financial volatility best according to the results of the statistical loss functions, which are preferred 26, 14 and 10 times, respectively. The GARCH model, on average has the worst financial volatility forecasting performance as it has resulted the highest value for 44 times. Comparing the results of each index, the EGARCH is the preferred model by the measures for eight stock indices S&P 500, CAC 40, Dow 30, DAX, TA125, IBOVESPA, S&P BSE SENSEX, FTSE 100 to forecast the financial volatility in a crisis like COVID-19 while GJR-GARCH is preferred by Nikkei 225 and NZ 50.

4. CONCLUSION

The main objective of this research project is to evaluate which GARCH model achieves the best predicted financial volatility closest to the realized financial volatility on large-cap stock indices during a crisis like COVID-19. The models examined in this study are GARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1) and PGARCH (1,1) on large-cap stock indices; S&P 500, CAC 40, Dow 30, Nikkei 225, DAX, TA125, IBOVESPA, S&P BSE SENSEX, FTSE 100 and NZ 50. The forecasting performance of the models is analyzed by using statistical loss functions; MSE, RMSE, QLIKE, R^2 Log and MAE. The best forecasting model is selected based on which model on average achieves the best financial volatility predictions. Also, this study examines whether there is a different impact on indices as they are representing nine different countries which represent 7 different regions.

The result of this paper concludes that EGARCH (1,1) on average predict the realized financial volatility best according to the statistical measures and then PGARCH (1,1) and GJR-GARCH (1,1). Hence, the characteristics of EGARCH, PGARCH and GJR-GARCH on capturing the asymmetry and leverage effect, obtain better predictions when forecasting financial volatility during a crisis on large-cap stock indices. Although the indices are from 9 different countries which represent 7 different regions, the COVID-19 crisis has been affected to all the ten stock indices almost equally as they are performing a large-cap, while NZ 50 and Nikkei 225 indices differ slightly as New Zealand and Japan are less affected by COVID-19 during the period 11.03.2020 – 10.03.2022.

Hence, we can conclude that, the best optimal model to forecast financial volatility on any large-cap stock indices during a crisis is EGARCH (1,1). This result can be different for mid-cap, small-cap, mega-cap, micro-cap and nano-cap stock indices according to the financial performance of the companies listed under these indices.

When considering limitations, the unavailability of historical data for large-cap stock indices representing the African region, such as JSE40 and EGX30, on Yahoo Finance may be due to access issues, political instability, or lack of transparency. Due to the broad scope of the study, the research was limited to examining only the forecasting performance of four models on ten large-cap stock indices for the period from 11th March 2015 to 10th March 2022.

Further extension, in future, I would like to explore the potential of ANN-GARCH, a machine learning technique that has been shown to outperform GARCH models in some studies, for financial volatility forecasting which is more appropriate for complex, nonlinear data with multiple inputs with more computational resources and more training data.

I would like to suggest a calculator and a dashboard using AI and ML models, which can present the best optimal model to forecast financial volatility for any stock / stock index where the investor can easily understand their risk and minimize their loss in a crisis.

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