MODELING POPULATION GROWTH OF SRI LANKAN CITIES

Panchadcharam Elango

Department of Mathematics, Faculty of Science, Eastern University, Sri Lanka

elangop@esn.ac.lk

ABSTRACT: This paper reports the findings of a study to test the population growth models to predict the population growth of two Sri Lankan cities; Batticaloa and Ampara. We study the population growth models; Malthusian's exponential growth model and logistic growth model and develop a new model to predict the population growth for Batticaloa and Ampara using the logistic growth model. We use the population data from the census population and the calculated populations by the Department of Census and Statistics of Sri Lanka.

Keywords: Mathematical Models, Population Growth, Malthus model, Logistic growth model.

1. INTRODUCTION

Population growth is one of the most concern problems in the world and the population projection is predicted by each and every country from the early ages. Population growth and its size of a country is a major factor for its economy and its policy. Most of the developed countries usually change their polices according to their population predictions. Many countries concern the consequences of the human population growth for its social and economic developments. There were many methodologies used to predict the population growth. The mathematical models play a major role to predict the population growth of the world. Mathematical models take many forms to predict the population growth such as dynamical systems, statistical models, models with differential equations and etc.

British economist Thomas Malthus (1766-1834) proposed a mathematical model on the paper, *An Essay on the Principle of Population as It Affects the Future Improvement of Society*. He proposed that the biological populations (including human ones) trends to increase at a rate of initial population size and developed an exponential growth model for human population (Malthus, 1956). This model is seen as the "grandfather" of all population models. Malthus believed that the human population grows geometrically while the food supplies grow arithmetically since it is limited by available land and technology. He stated that the "laws of nature" dictate that a population can never increases beyond the food supplies necessary to support it (Murray, 2001). However the real data showed that the Malthus model is unreasonable over a long period of time. Some populations grow exponentially which are not too large. In the case of human population, normally the population is too large, individual members compete with one another for food, living space and other natural resources. Malthus model may not be able to use to predict the population for a long period of time so it became unrealistic for the prediction of human population.

Dutch mathematical biologists Pierre-Francois Verhulst (1804-1849) proposed another population growth model in the 1840's which is not only depends on the population size but also depends on its upper limit, called the carrying capacity of the environment (Verhulst, 1838). He developed this model using the Malthus' exponential growth model with an addition to a population function. This model was rediscovered and popularized in the 1920's by Pearl and Reed (Pearl and Reed, 1924). The model proposed by Verhulst is referred to as the logistic growth model and it is widely established in many fields of mathematical modeling (Banks, 1994). The logistic growth model has three parameters, the initial population, the growth rate and the carrying capacity of the environment which to be used to fit the real data. These were used by Pearl (Pearl, 1925) to fit the census population data for various countries.

2. METHODOLOGY

The population growth model for the population P which is proposed by Malthus was

$$\frac{dP}{dt} = (b-d)P,$$

where $(b-d)$ is the per capita growth rate. The per capita growth rate r is calculated by

$$r = \frac{dP}{dt}\frac{1}{P}.$$

The solution of the Malthus differential equation model by the separation of variable becomes

$$P(t) = P_0 e^{rt},$$

where $P(t)$ is the population at time t, P_0 the initial population and $r = b - d$ the per capita growth rate.

Assuming that the environment has an intrinsic carrying capacity, K, Verhulst proposed the following model:

$$\frac{dP}{dt} = (b-d)Pf(P),$$

,

where $f(P)$ is the population growth function which should satisfy that $f(0) = 1$, that is the population grows exponentially with the growth rate $(b-d)$ when P is small. This is actually the Malthus exponential growth model. The function should also satisfy that $f(K) = 0$, that is the population growth becomes zero when the population reached its maximum, which is called the carrying capacity K of the environment. Further, $f(P) < 0$ for $P > K$, that is the population should decrease when its size exceeds the carrying capacity of the environment. To satisfy all these conditions, he proposed the simplest population growth function $f(P)$ as

$$f(P) = \left(1 - \frac{P}{K}\right).$$

Then the Verhulst' population growth model can be written as

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right),$$

where $r = b - d$. To solve this model we use the separation of variables method and we get

$$\int \frac{dP}{P\left(1 - \frac{P}{K}\right)} = \int r\,dt.$$

Resolving into partial fractions, we get

$$\int \frac{dP}{P} + \int \frac{dP}{K - P} = \int r\,dt.$$

Solving this differential equation and substituting the initial condition, $t = 0, P = P_0$, we finally get Verhulst's population growth model to predict the population $P(t)$ at time t as

$$P(t) = \frac{KP_0 e^{rt}}{K + P_0(e^{rt} - 1)}.$$

As $t \to \infty$, we see that $P(t) = K$, the maximum population which we call the carrying capacity of the environment. Verhulst call this model as the logistic growth model. The logistic model was widely accepted in single species population growth situations by biologists. The important aspect about the logistic model is that it is particularly in algebraic simplicity and in convenient form to take qualitative dynamic behavior in populations.

The logistic growth model has three parameters, P_0, r and K which to be used to fit the real data. Calculating the carrying capacity K is the challenging part and the weakness of this model. There is no accurate methodology exists to find the carrying capacity since the carrying capacity is connected with many factors including environment impacts and natural resources (Joel E. Cohen, 1995). There were many different approaches made and predicted the carrying capacity of the world (Russell, 2003). We calculate the carrying capacity K, from the following equation which is directly derived from the model

$$K = \frac{P(t) \times P_0(e^{rt} - 1)}{P_0 e^{rt} - P(t)}$$

This equation is theoretically correct so we use it to predict the carrying capacity of the Sri Lankan cities. Since the cities have limited variables such as food supply, living space, health facilities and employment opportunities which influence the carrying capacity, our calculations will give theoretically correct value for K.

3. RESULTS AND DISCUSSION

We use this model to predict the population of Sri Lankan cities. In this paper, we predict the population of the Batticaloa and Ampara by the Verhulst logistic growth model. For the calculation of the parameters, we use the two census populations on 1971, 1981 and the calculated population on 2001 by the department of census and statistics. For verification of our model, we use the census population of the year 2011. The following table displays the populations and per capita growth rate for these years.

Table 1. Batticaloa Population

Year	Population	Per capita growth rate
1971	256,721	0.033
1981	330,333	0.027
2001	486,447	0.019
2011	526,567	

Using the formula for the carrying capacity,K we get 1,239,100 and 1,430,900 are the carrying capacities for 1981 and 2001 respectively. The increase in carrying capacity is acceptable as the natural resources usually increase in each year.

To finalize the carrying capacity K for our model, we use another calculated population of the year 2015 by the department of census and statistics which is the population of the year 2015 as 541,000. Now we use the Verhulst model and calculate the population of the year 2015 by using the above two values of K. By comparing our value with the department of census and statistics value, we can approximate a value for K and finalize the model. So we take the initial population P_0 as the population of the year 2001, the per capita growth rate $r = 0.019$ and the values of $K =$ 1,239,100 and $K =$ 1,430,900 and calculate the population of the year 2015.

When K = 1,239,100 we get the population as 566,870 and when K = 1,430,900 we get the population as 575,105. Therefore, we can choose K = 1,239,100 since it predict the population of the year 2015 is closer to the calculated population by the department of census and statistics of this year. To predict the population closer to the calculated population and for simplification, we choose the carrying capacity of Batticaloa as K = 1,000,000. Using this calculated value for K we formulated the population growth model for Batticaloa as

$$P(t) = \frac{1{,}000{,}000\, P_0 e^{rt}}{1{,}000{,}000 + P_0(e^{rt} - 1)}$$

with the initial population P_0 and the per capita growth rate r.

Now we use the above model to predict the Batticaloa population for the year 2011 and compare our result with the census population of the year 2011 which is given in the table-1, above. We use the initial year as 2001 and its population as the initial population $P_0 = 486{,}447$ and the per capita growth rate $r = 0.019$. Our model's predicted population is 533,890 while the census population is 526,567. The percentage error in our calculation is 01.39% and it is acceptable rate since the 2001 population is also a calculated population by the department of census and statistics by some statistical methods. If we have an actual census population for the year 2001, then we can get more accurate population prediction.

Now we consider another city and develop a model for population prediction. For this modeling, we consider Ampara and formulate a model to predict the population growth for Ampara using the Verhulst logistic growth model. For the derivation of the model, we use the census populations on 1971, 1981 and 2011 and the calculated population on 2001 by the department of census and statistics. The following table displays the populations and per capita growth rate for these years.

Table 2. Ampara Population

Year	Population	Per capita Growth rate
1971	272,605	0.031
1981	388,970	0.038
2001	592,997	0.021
2011	649,402	

When we take the two census population of the year 1971 and 1981 and the per capita growth rate $r = 0.031$, we obtain the carrying capacity $K = 2{,}228{,}300$. When we take the populations of the calculated years 2001 and 2015, which was calculated by the department of census and statistics as 728,963, with the per capita growth rate $r = 0.021$ we get the carrying capacity $K = 2{,}425{,}300$. By considering other census population and by other calculations, we take $K = 2{,}000{,}000$. Therefore, we propose the population prediction model for Ampara is

$$P(t) = \frac{2{,}000{,}000\, P_0 e^{rt}}{2{,}000{,}000 + P_0(e^{rt} - 1)}$$

Now we use our model to predict the Ampara population for the year 2011 and compare our result with the census population given in the table-2 above. We use the initial year as 2001 and the per capita growth rate $r = 0.021$. Our model's predicted population is 684,160 while the census population is 649,402. The percentage error in our calculation is 05.35%. The reason for this variation may be the sudden low per capita growth rate in the year 2001 by some socio economic factors. Also the population for the year 2001 is also a calculated value by the department of census and statistics by some statistical methods, there may be some calculation errors. To minimize the error percentage, we need more census population which was not done to the country due to the 30 years of the ethnic war in the country.

4. CONCLUSION

We modified the Verhulst logistic population growth model and derived the following population growth model to predict the population of Batticaloa district as

$$P(t) = \frac{1{,}000{,}00\, P_0 e^{rt}}{1{,}000{,}000 + P_0(e^{rt} - 1)}$$

where $P(t)$ is the population for any year t, P_0 is the initial population and r is the per capita growth rate. This model is best to predict the population after the year 2011 with the initial population P_0 as the population of the year 2011, the last census year and with the per capita growth rate $r =$ 0.0123 (for the year 2011). Using this model, we can approximately predict the population of Batticaloa after the year 2011 until the next census. Using the next census population, we can get the new per capita growth rate and the new carrying capacity of Batticaloa district.

The second model we developed in this paper is to predict the population growth of Ampara district as

$$P(t) = \frac{2{,}000{,}000\, P_0 e^{rt}}{2{,}000{,}000 + P_0(e^{rt} - 1)}.$$

This model is also best to predict the population after the year 2011, the last census year. Here we take P_0 as the initial population of the year 2011 and the per capita growth rate $r =$ 0.0123 (for the year 2011). Using this model, we can approximately predict the population of Ampara after the year 2011 until the next census.

Using the same methodology and the population data, we can develop the population growth models for other cities of Sri Lanka. The one problem we faced during the development of the models was the lack of accurate population data since the census did not done for 3 times because of the civil war in Sri Lanka.

5. **REFERENCES**

BANKS, R.B, (1994), Growth and Diffusion Phenomena, Mathematical Frame - works and Applications, Springer, Berlin.

JOEL E. COHEN, (1995), Population growth and earth's carrying capacity, Science, New Series 269(5222), pp.341-346.

MALTHUS, T. R, (1956), Mathematics of population and food, The World Mathematics, New York, pp. 1189-1199.

MURRAY, J.D, (2001), Mathematical Biology I, An Introduction, Springer-Verlag.

PEARL R, (1925), The biology of population growth, Alfred A Knopf, New York.

PEARL, R. and REED, L. J, (1924), The growth of human population, in Studies in Human Biology.

RUSSELL HOPFENBERG, (2003), Human Carrying Capacity Is Determined by Food Availability, Population and Environment, 25(2), pp.109-117.

VERHULST, P. F, (1838) Notice sur la loi que la population poursuit dans son Accroissement, Correspondance mathematique et physique 10, 113121.