

Elliptic Triangulations of Spheres

P. Elango

Department of Mathematical Sciences, South Eastern University of Sri Lanka,
Corresponding Author: elango@seu.ac.lk

The degree of a vertex x in a triangulation T of a sphere is the number of triangles $\Delta_1, \Delta_2, \dots$, which contain x and is denoted by $d = d(x)$. A triangulation T is said to be elliptic if it does not contain any point with degree greater than 6, that is, $d(x) < 6$ for every $x \in T$. We used Euler's equation to get

$$3\alpha_3 + 2\alpha_4 + \alpha_5 - 2\alpha_6 - (m - 6)\alpha_m = 12,$$

which reduces to

$$3\alpha_3 + 2\alpha_4 + \alpha_5 = 12 \text{ in the elliptic case. There are}$$

19 nonnegative solutions $((\alpha_3, \alpha_4, \alpha_5))$ for this equation.

We call $(\alpha_3, \alpha_4, \alpha_5)$ is the type of the triangulation T . It has been shown that for each of the solution $((\alpha_3, \alpha_4, \alpha_5))$ there exist a triangulation T and a non negative integer $N = N(\alpha_3, \alpha_4, \alpha_5)$ with the property

$$E_3(T), E_4(T), \dots, E_6(T) = (\alpha_3, \alpha_4, \alpha_5, N).$$

Our main aim was to find, for each of the 19 types of triangulations, all possible values of $N = N(\alpha_3, \alpha_4, \alpha_5)$. We describe various methods to construct elliptic spherical triangulations such as the mutant, productive and self-reproductive configurations, the fullering constructions and the glueing of patches method.

We remark here that some non-existence results on triangulations have been obtained by Grunbaum, Eberhard, and Bruckner have determined the minimum values of N such that

the triangulations of type $(\alpha_3, \alpha_4, \alpha_5, N)$ exist for each of the 19 nonnegative solutions $(\alpha_3, \alpha_4, \alpha_5)$.

Key Words: Polygon, Triangulations, Patches

the triangulations of type $(\alpha_3, \alpha_4, \alpha_5, N)$ exist for each of the 19 nonnegative solutions