STANDARD METHOD TO COMPARE THE COMBINED QUALITY MONITORING SCHEMES USING AVERAGE RUN LENGTH PROPERTIES

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Introduction

In quality monitoring the mean and variance were monitored separately by using Shewhart, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts.Gan (1997) emphasized that quality monitoring is really a bivariate problem, which should not be dealt with as two separate univarite problems. Monitoring the mean or variance separately might deceive quality control engineers into making inferences concerning the mean or the variance chart without making reference to the other. Therefore joint monitoring of process mean and variance became necessity. Gan (1997) discussed two types of Shewhart combined schems one with rectangular control region and other with elliptical control region. The CUSUM and EWMA charts can be combined for joint monitoring and it is given Gan (1995). Max charting scheme for joint monitoring was proposed by Chen and Cheng (1998). Max EWMA and EWMA-semicircle schemes were proposed by Chen et al in 2001 and 2004 respectively. For comparing the performances of these combined joint monitoring schemes, a standard method is required and it is proposed in this paper. The average run length (ARL) properties of the combined schemes were used for comparing the performances.

Methodology

The ARL is the average number of samples taken until an out-of-control signal is issued in quality control schemes. The control limits for a quality control schemes are usually determined based on the ARL consideration and the performances of the different quality control charting schemes are compared by using the out of control ARLs for particular shift in process mean. The scheme which gives lowest out-of-control ARL when there is a shift in process mean is considered to be the best scheme. The same technique is followed to compare the performance of the control charting schemes for the process variance. Let X_{tj} denote a certain quality characteristic of a process where t is the sample number, j is the j^{th} unit of the sample and j = 1, 2, ..., n. It is assumed that X_{tj}'s are independently and identically normally distributed random variables with mean μ_0 and standard deviation σ_0. For a standard comparison the sample mean $\bar{X}_t$ and sample variance S_t^2 can be standardized as $U_t = \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}}$ and $V_t = \Phi^{-1}\left[\mathrm{H}\left(\frac{(n-1)S_t^2}{\sigma_0}; n-1\right)\right]$ respecyively where $\mathrm{H}\left(\frac{(n-1)S_t^2}{\sigma_0}; n-1\right) = \mathrm{H}(w; v) = \mathrm{P}(W \le w) for W \sim \chi_v^2$, the chi-square distribution with v degrees of freedom. For each sample there will be a standardized mean U_t and standardized variance V_t. Any schemes for monitoring U_t will have the Upper Control limit (UCL_M) and lower control limit (LCL_M) with in control ARL of ARL_M. In the same way any schemes for monitoring V_t will have the Upper Control limit (UCL_V) and lower control limit (LCL_V) with in control ARL of ARL_V. The combined charting schemes for monitoringU_t and $V_t$will have the in-control ARL of ARL_c where $\frac{1}{ARL_C} = \frac{1}{ARL_M} + \frac{1}{ARL_V}$.This technique can be

programmed in any advance statistical software and comparison of the combined charting schemes become easy and standard. Any studied schemes can be compared among them using the index

$$E_{S,\Delta,\delta} = 100 - \left(\frac{ARL_{Out-of-control} - ARL_{Min}}{ARL_{Min}}\right) X100$$

Where

$E_{S,\Delta,\delta} = Efficincy\ of\ S\ \ scheme\ for\ \Delta\ shift\ in\ mean\ and\ \delta\ shift\ in\ variance$

$ARL_{Out-of-control} = Out - of - control\ ARL\ for\ \Delta\ shift\ in\ mean\ and\ \delta\ shift\ in\ variance\ for\ S\ Scheme$

$ARL_{Min} = Minimum\ Out - of - ontrol\ ARL\ for\ \Delta\ shift\ in\ mean\ and\ \delta\ shift\ in\ variance\ among\ the\ schemes.$

Discussion and Conclusion

A sample comparison of combined monitoring schemes is shown in Figure 1. for a simulated data. In this comparison threeShewhart combined schemes with rectangular control region (SS_r), elliptical control region (SS_e) and distance control region (SD) are compared for efficiency. The comparison of efficiency of different combined schemes is very obvious in this proposed method.

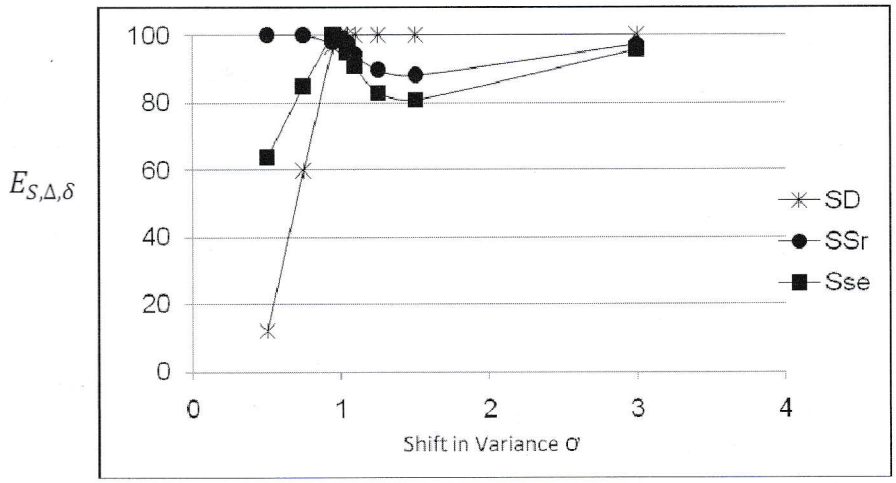

Figure 1: Efficiency Index $E_{S,\Delta,\delta}$ for Different shifts in δ for ARL 250

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