Real Time Traffic Control to Optimize Waiting Time of Vehicles at A Road Intersection

Yogeswary Raviraj[1], W.B.Daundasekera[2]

[1](Department of Mathematical Sciences, Faculty of Applied Sciences,
South Eastern University of Sri Lanka, Sri Lanka)
[2](Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka)
Corresponding Author: Yogeswary Raviraj

Abstract : *This paper presents a real time non-linear quadratic programming model to minimize the aggregate delay time of vehicles at each lane by minimizing number of vehicles at each lane at the signalized intersection. One of the most important efficiency and easily getting factors of traffic signal control is the number of vehicles in a queue at the signalized intersection. The initial number of vehicles at each lane at the intersection is counted by camera which is the most accurate method.The model is developed to minimize the number of vehicles from cycle to cycle. This proposed model includes inter green signal time which is one of the key factors compared to other existing models proposed in the past research. This model also incorporates restrictions for upper and lower bounds for green signal time and cycle time allocation which leads to accurate and appropriate allocation for green signal time. The proposed model is solved by the method of sequential quadratic programming coded in MATLAB environment.*

Keywords–*aggregate delay time*, *nonlinear sequential quadratic programming*, *optimization*, *real time*, *traffic signal*

I INTRODUCTION

In any country traffic congestion wastes a huge amount of the national income for fuel and traffic – related environmental and socio economic problems. Traffic intersections contribute a lot to this traffic congestion. Signalized intersections are crucial points in the highway systems. The past researches [1] reveal that the real time traffic control strategy gives a better traffic control system than the fixed-time (off-line) traffic signal control strategy. Real time traffic signal control optimization is complex due to its nature. Delay [2] and the number of vehicles waiting are the important measures of effectiveness for signalized intersections. This research considers the oversaturation level at road intersections, where the intersections considered have four signals for a particular minimum cycle time, and the green time is allocated for all four signals.

Our objective of this research is to formulate a mathematical model to minimize aggregate delay [3] time of vehicles and the total number of vehicles which are waiting in the lanes of a road intersection due to red signal by allocating sufficient amount of green time for each signal and cycle time. To maintain the feasibility, the upper bound for cycle time and the upper bound of number of vehicles waiting are adjusted according to the incoming flow rate and outgoing flow rate of vehicles. The real time data is calculated by using cameras [4] installed in every lane in the road intersection. This model is solved by sequential quadratic programming algorithm coded in MATLAB environment [5].

II METHODOLOGY

In this research we consider a signalized isolated intersection with four lanes namely Lane j, $j = 1,2,3,4$. We divide this intersection into four stages: in Stage 1, the green signal will be on for Lane 1, where the vehicles which are waiting at the Lane 1 can move into other three lanes through the intersection as shown in the figure below:
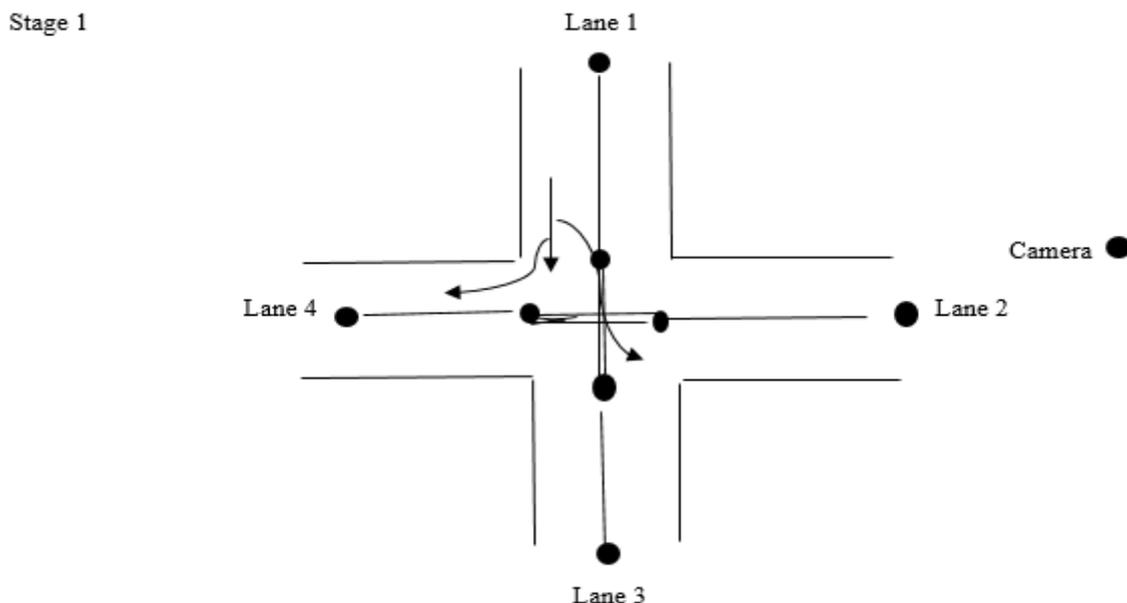
Fig.1.Green signal for Lane 1

Also, when the green signal is on for another lane the vehicles waiting in that lane will proceed to other lanes in a similar manner as described above in the Fig.1.

2.1 Formulation of the Model

2.1.1 Notations
Lane j: j^{th} lane at the road intersection, j=1, 2, 3, 4,
$N_j(k)$: Number of vehicles in Lane j at cycle k,
Ig: inter green time,
W_j: Weighting parameter of Lane j, j=1, 2, 3, 4,
$t_j(k)$: Allocated green time for the signal for Lane j, j=1, 2, 3, 4 at cycle k,
$f_j(k)$: In coming flow rate of vehicles for the Lane j, j=1, 2, 3, 4 at cycle k,
$s_j(k)$: Outgoing flow rate of vehicles for the Lane j, j=1, 2, 3, 4 at cycle k,
$(t_j(k))_{min}$: Minimum green time for the signal for Lane j, j=1, 2, 3, 4, at cycle k,
$(t_j(k))_{max}$: Maximum green time for the signal for Lane j, j=1, 2, 3, 4, at cycle k,
$C(k)$: Cycle time at cycle k,
CT_{min}: Minimum cycle time,
CT_{max}: Maximum cycle time,
$D_j(k+1)$: Aggregate delay time of vehicles in cycle k at Lane j, j=1, 2, 3, 4.

2.1.2 Formulation
In Stage 1, the aggregate delay time of vehicles for $(k+1)^{th}$ cycle (aggregate delay time of vehicles at the end of kth cycle) is calculated by the aggregate delay time of vehicles in the Lane 1 during green signal is on and after the green signal is off which is represented by the shaded area of Stage 1 in Fig.2.

In Stage 2, the aggregate delay time of vehicles for $(k+1)^{th}$ cycle (aggregate delay time of vehicles at the end of kth cycle) is calculated by the aggregate delay time of vehicles in the Lane 2 before green signal is on, during green signal is on and after the green signal is off which is represented by the shaded area of Stage 2 in Fig.2. Similarly for the other two stages, the total delay time for $(k+1)^{th}$ cycle is illustrated in the figure below
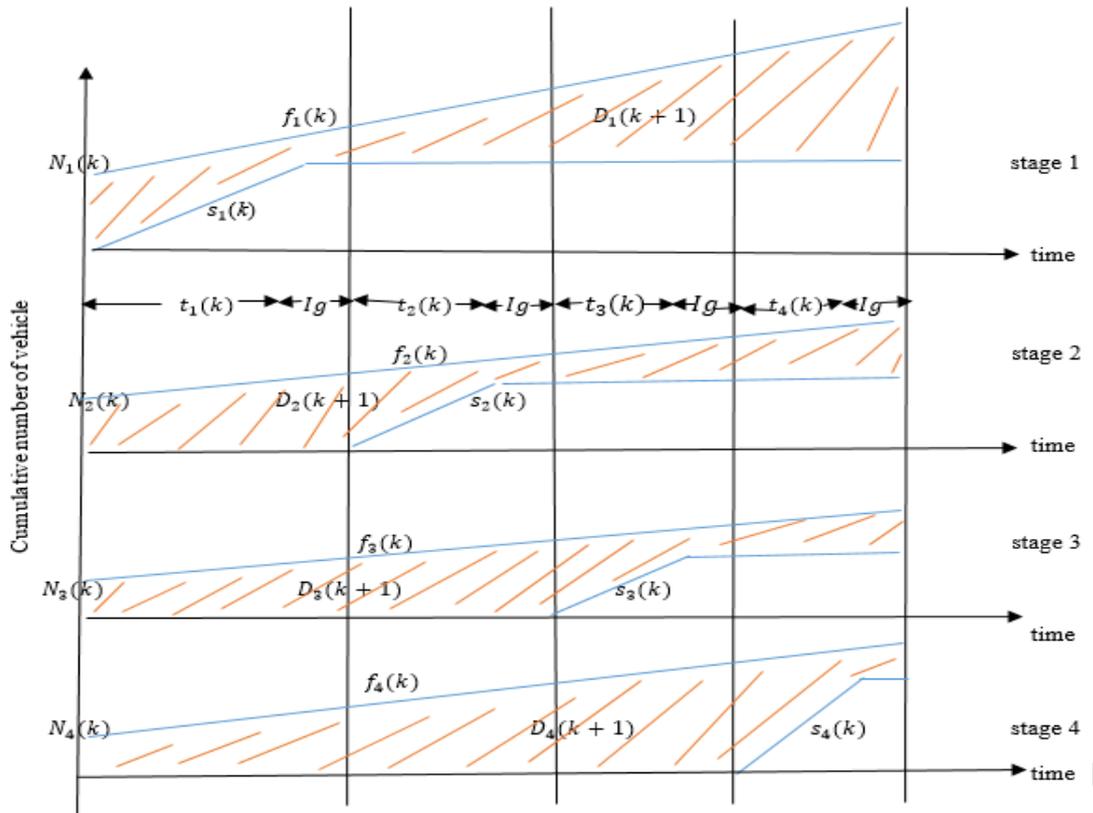
Fig.2.Number of vehicle and delay of four stages

Aggregate delay time for vehicles at each stage at the end of cycle *k* is calculated from the area of the shaded region in the Fig.2 as given below:

$$D_1(k+1) = \frac{1}{2}\left[2N_1(k)C(k) + f_1(k)C(k)^2 - s_1(k)\left(t_1(k)\right)^2 - 2s_1(k)t_1(k)(4Ig + t_2(k) + t_3(k) + t_4(k))\right] \quad (1)$$

$$D_2(k+1) = \frac{1}{2}\left[2N_2(k)C(k) + f_2(k)C(k)^2 - s_2(k)\left(t_2(k)\right)^2 - 2s_2(k)t_2(k)(3Ig + t_3(k) + t_4(k))\right] \quad (2)$$

$$D_3(k+1) = \frac{1}{2}\left[2N_3(k)C(k) + f_3(k)C(k)^2 - s_3(k)\left(t_3(k)\right)^2 - 2s_3(k)t_3(k)(2Ig + t_4(k))\right] \quad (3)$$

$$D_4(k+1) = \frac{1}{2}\left[2N_4(k)C(k) + f_4(k)C(k)^2 - s_4(k)\left(t_4(k)\right)^2 - 2s_4(k)t_4(k)Ig\right] \quad (4)$$

The number of vehicles in four lanes at the end of the cycle *k* is given by the following four equations. First equation represents number of vehicles in Lane 1 at the end of cycle *k* (or beginning of cycle *k*+1) which is equivalent to the total of the number of vehicles waiting at the beginning of cycle *k* (from camera readings), incoming number of vehicles into the Lane 1 during green signal time, inter signal green time and red signal time and excluding outgoing number of vehicles during green signal time for the Lane 1. Similarly, the number of vehicles in Lane 2, Lane 3 and Lane 4, at the end of cycle *k* (for cycle *k*+1) are given by the remaining three equations respectively:

$$N_1(k+1) = N_1(k) + (t_1(k) + Ig)f_1(k) - (t_1(k))s_1(k) + (t_2(k) + Ig)f_1(k) +$$
$$(t_3(k) + Ig)f_1(k) + (t_4(k) + Ig)f_1(k) \quad\quad\quad (5)$$

$$N_2(k+1) = N_2(k) + (t_2(k) + Ig)f_2(k) - (t_2(k))s_2(k) + (t_1(k) + Ig)f_2(k) +$$
$$(t_3(k) + Ig)f_2(k) + (t_4(k) + Ig)f_2(k) \quad\quad\quad (6)$$

$$N_3(k+1) = N_3(k) + (t_3(k) + Ig)f_3(k) - (t_3(k))s_3(k) + (t_1(k) + Ig)f_3(k) +$$
$$(t_2(k) + Ig)f_3(k) + (t_4(k) + Ig)f_3(k) \quad\quad\quad (7)$$

$$N_4(k+1) = N_4(k) + (t_4(k) + Ig)f_4(k) - (t_4(k))s_4(k) + (t_1(k) + Ig)f_4(k) +$$
$$(t_2(k) + Ig)f_4(k) + (t_3(k) + Ig)f_4(k) \quad\quad\quad (8)$$

Lower and upper bounds of green signal time:
$$(t_j(k))_{min} \leq t_j(k) \leq (t_j(k))_{max} \,,\; j = 1,2,3,4 \tag{9}$$

Cycle time of cycle k is given by the total green signal time and total inter green signal time:
$$(t_1(k) + Ig) + (t_2(k) + Ig) + (t_3(k) + Ig) + (t_4(k) + Ig) = C(k) \tag{10}$$

Lower and upper bounds of cycle time:
$$CT_{min} \leq C(k) \leq CT_{max} \tag{11}$$

When twice the number of outgoing vehicles during green signal time in a lane at a given a cycle is less than the number of vehicles in that lane at the end of that cycle, it is defined as oversaturation condition. Each of the following four inequalities represents the oversaturation condition for each of the four lanes during cycle k:
$$2s_1(k)t_1(k) \leq N_1(k+1) \tag{12}$$
$$2s_2(k)t_2(k) \leq N_2(k+1) \tag{13}$$
$$2s_3(k)t_3(k) \leq N_3(k+1) \tag{14}$$
$$2s_4(k)t_4(k) \leq N_4(k+1) \tag{15}$$
The incoming number of vehicles into the lane during a cycle time is less than or equal to the outgoing number of vehicles from the lane during green signal time for that lane is also considered as a condition. Each of the following four inequalities represents that condition for each of the respective four lanes during cycle k:
$$f_1(k)C(k) \leq s_1(k)t_1(k) \tag{16}$$
$$f_2(k)C(k) \leq s_2(k)t_2(k) \tag{17}$$
$$f_3(k)C(k) \leq s_3(k)t_3(k) \tag{18}$$
$$f_4(k)C(k) \leq s_4(k)t_4(k) \tag{19}$$

2.2 Flow Chart of the Model and the Method of Solution
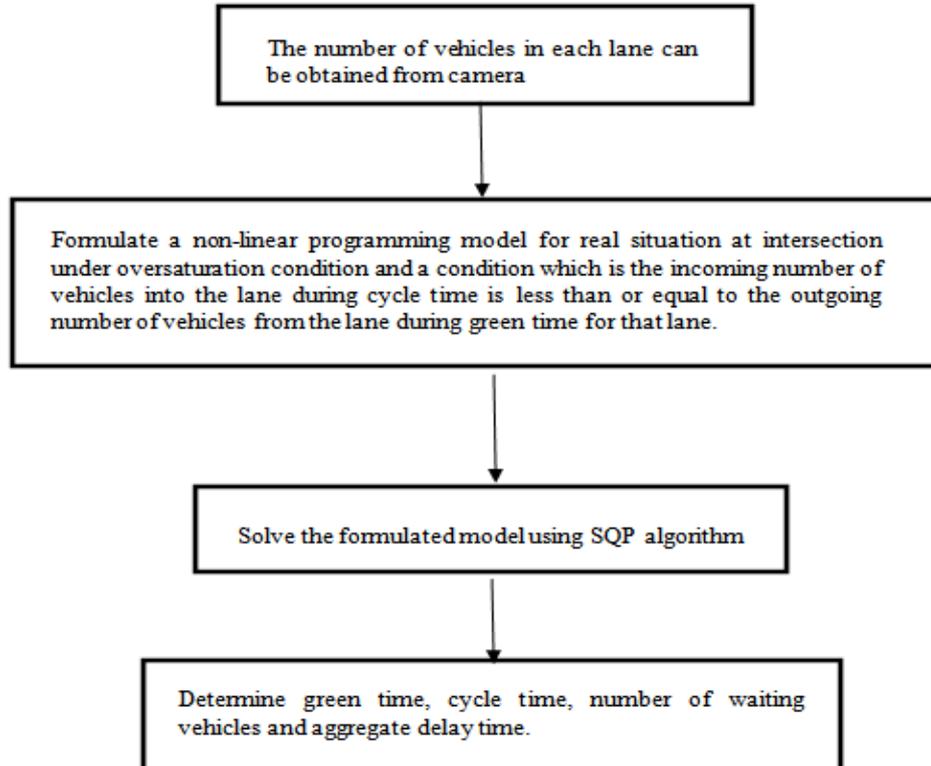

Fig.3. Flow chart for the model and the method of solution

III   OPTIMIZE GREEN TIME

Objective is to minimize the total number of vehicles waiting at the intersection subject to the oversaturation condition, the delay, additional condition and some constraints related to the traffic signal control problem, which are described above, are combined into the model formulation and is illustrated below to calculate the duration of the green signal time on at the beginning of the cycle k:

Minimize $Z = \sum_{j=1}^{4} W_j N_j(k+1)$

Subject to

$N_1(k+1) = N_1(k) + (t_1(k) + Ig) * f_1(k) - (t_1(k)) * s_1(k) + (t_2(k) + Ig) * f_1(k) + (t_3(k) + Ig) * f_1(k) + (t_4(k) + Ig) * f_1(k),$

$N_2(k+1) = N_2(k) + (t_2(k) + Ig) * f_2(k) - (t_2(k)) * s_2(k) + (t_1(k) + Ig) * f_2(k) + (t_3(k) + Ig) * f_2(k) + (t_4(k) + Ig) * f_2(k),$

$N_3(k+1) = N_3(k) + (t_3(k) + Ig) * f_3(k) - (t_3(k)) * s_3(k) + (t_1(k) + Ig) * f_3(k) + t_2(k) + Ig) * f_3(k) + (t_4(k) + Ig) * f_3(k),$

$N_4(k+1) = N_4(k) + (t_4(k) + Ig) * f_4(k) - (t_4(k)) * s_4(k) + (t_1(k) + Ig) * f_4(k) + (t_2(k) + Ig) * f_4(k) + (t_3(k) + Ig) * f_4(k),$

$2s_1(k)t_1(k) \le N_1(k+1),$
$2s_2(k)t_2(k) \le N_2(k+1),$
$2s_3(k)t_3(k) \le N_3(k+1),$
$2s_4(k)t_4(k) \le N_4(k+1),$
$f_1(k)C(k) \le s_1(k)t_1(k),$
$f_2(k)C(k) \le s_2(k)t_2(k),$
$f_3(k)C(k) \le s_3(k)t_3(k),$
$f_4(k)C(k) \le s_4(k)t_4(k),$

$D_1(k+1) = \frac{1}{2}\left[2N_1(k)C(k) + f_1(k)C(k)^2 - s_1(k)\big(t_1(k)\big)^2 - 2s_1(k)t_1(k)(4Ig + t_2(k) + t_3(k) + t_4(k))\right] \ge 0,$

$D_2(k+1) = \frac{1}{2}\left[2N_2(k)C(k) + f_2(k)C(k)^2 - s_2(k)\big(t_2(k)\big)^2 - 2s_2(k)t_2(k)(3Ig + t_3(k) + t_4(k))\right] \ge 0,$

$D_3(k+1) = \frac{1}{2}\left[2N_3(k)C(k) + f_3(k)C(k)^2 - s_3(k)\big(t_3(k)\big)^2 - 2s_3(k)t_3(k)(2Ig + t_4(k))\right] \ge 0,$

$D_4(k+1) = \frac{1}{2}\left[2N_4(k)C(k) + f_4(k)C(k)^2 - s_4(k)\big(t_4(k)\big)^2 - 2s_4(k)t_4(k)Ig\right] \ge 0,$

$(t_1(k) + Ig) + (t_2(k) + Ig) + (t_3(k) + Ig) + (t_4(k) + Ig) = C(k),$

$(t_j(k))_{min} \le t_j(k) \le (t_j(k))_{max}, \quad j = 1,2,3,4,$

$CT_{min} \le C(k) \le CT_{max}.$

The cycle time is equal to the sum of the total green signal time and total inter green time. Each green time value has a minimum value $((t_j(k))_{min})$ and a maximum value $((t_j(k))_{max})$ which are fixed for a cycle.

In order to maintain the feasibility, the sum of the total minimum green signal time values and inter green signal time values is assumed to be greater than or equal to CT_{min} and also, the sum of the total maximum green signal time values and inter green signal time values is assumed to be less than or equal to CT_{max}.

The objective function consists of waiting parameters $W_j, j = 1, 2, 3, 4$ assigned to each lane. The default value of $W_j, j = 1, 2, 3, 4$ is assigned to 1. The objective function can be optimized by selecting different waiting parameters W_j according to different criteria: lane priority, emergency vehicle passing etc.

To optimize green time for each signal we apply sequential quadratic programming algorithm implemented in the MATLAB optimization toolbox for the above nonlinear programming problem.

IV   RESULTS AND DISCUSSION

The method is tested to a hypothetical data set:

Distance between upstream camera and downstream camera installed in any lane is 125 m. If we assume that the average length of a small vehicles is approximately 5 m, then the maximum number of vehicles in a lane within 125 m is 25.

Incoming flow rates of vehicles for the lanes are fixed over the cycles given by
$f_1 = 0.05$ vehicles /sec., $f_2 = 0.1$ vehicles/sec., $f_3 = 0.05$ vehicles/sec., $f_4 = 0.1$ vehicles/sec.

Outgoing flow rates of vehicles for the lanes are fixed over the cycles given by
$s_1 = 0.5$ vehicles/sec., $s_2 = 0.5$ vehicles/sec., $s_3 = 0.5$ vehicles/sec., $s_4 = 0.5$ vehicles/sec.

Inter green signal time is given by Ig=1 sec.
Weighting parameters of the lanes are given by W_j=1 for j=1, 2, 3, 4.

Simulation results and corresponding graphs are given in the following Table 1, Fig.4 and Fig.5 for scenario 1:
From camera readings: $N_1(1) = 20$, $N_2(1) = 23$, $N_3(1) = 21$, $N_4(1) = 25$

Table 1. Cycles and the optimum feasible results for scenario 1

Cycle k	Number of vehicles in lanes at the beginning of the cycle $N_j(k), j = 1,2,3,4$				Green signal time (sec.) $t_j(k), j = 1,2,3,4$				Cycle time (sec.) $C(k)$	Aggregate delay time at the end of the cycle (sec.) $D_j(k+1)$, $j = 1,2,3,4$	Number of vehicles in lanes at the end of the cycle $N_j(k+1)$, $j = 1,2,3,4$			
1	20	23	21	25	16	21	17	22	80	1184, 1609, 1564, 2188	16	21	17	22
2	16	21	17	22	13	19	14	19	69	817, 1255, 1096, 1656	13	18	13	19
3	13	18	13	19	11	16	11	17	59	560, 924, 719, 1214	10	16	10	16
4	10	16	10	16	08	14	08	14	48	362, 659, 458, 827	08	14	08	14
5	08	14	08	14	07	12	07	12	42	245, 508, 319, 634	07	12	07	12
6	07	12	07	12	06	10	06	10	36	185, 377, 239, 467	06	11	06	11
7	06	11	06	11	05	10	05	10	34	154, 317, 197, 402	05	09	05	09
8	05	09	05	09	04	08	04	08	28	108, 215, 136, 271	04	08	04	08
9	04	08	04	08	04	07	04	07	26	73, 181, 99, 226	03	07	03	07
10	03	07	03	07	03	06	03	06	22	47, 133, 64, 166	03	06	03	06
11	03	06	03	06	03	05	03	05	20	42, 106, 57, 131	03	06	03	06
12	03	06	03	06	03	05	03	05	20	42, 106, 57, 131	03	06	03	06

In the Table 1 given above, the results of cycle 11 and cycle 12 are the same. If this process continues into more cycles, the results will be the same as the results of cycle 11. Because of oversaturation situation some vehicles are still waiting in each lane in the last cycle (cycle 11).
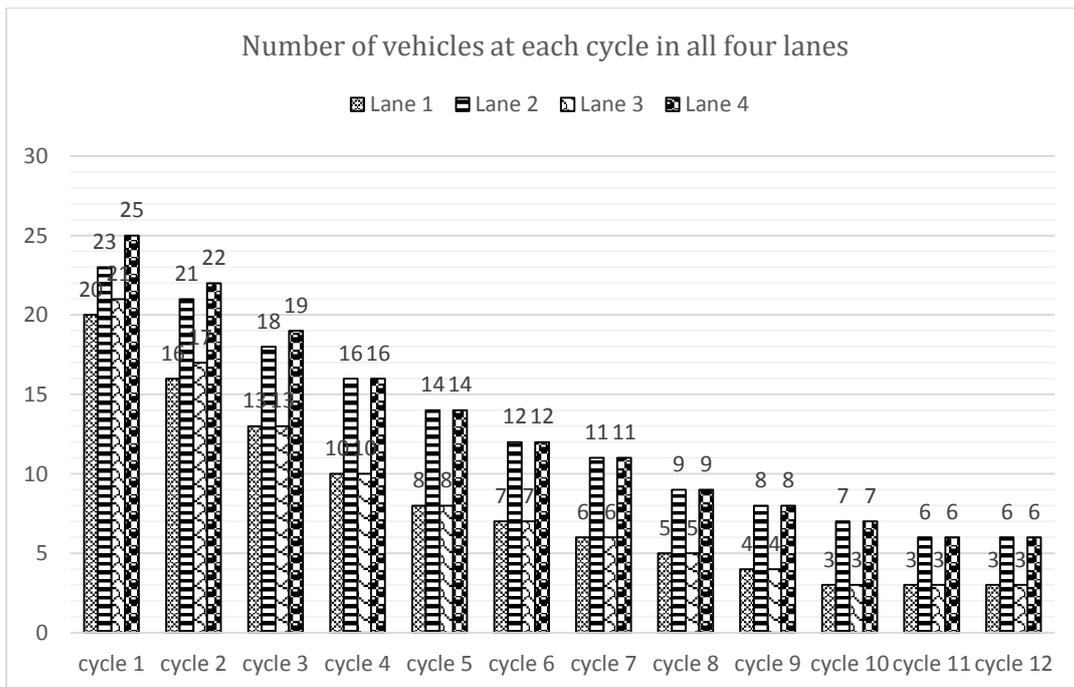

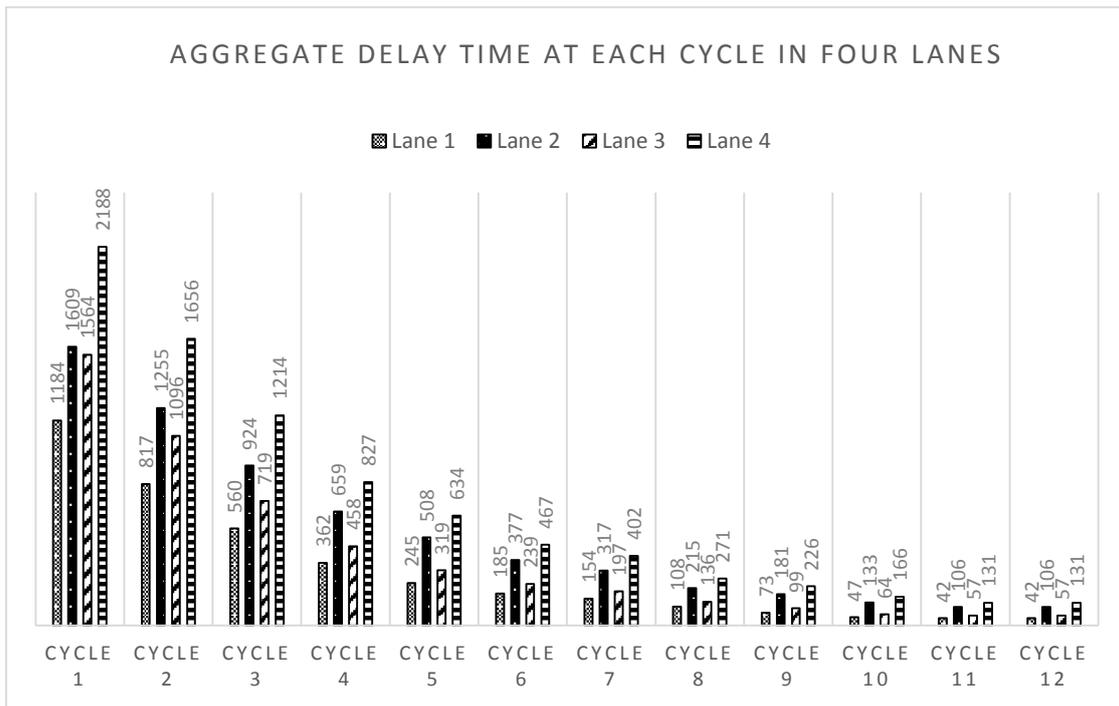**Fig.4.Number of vehicles at each cycle in Lane 1, Lane 2, Lane 3 and Lane 4 at scenario 1**

Fig.5. Aggregate delay time at each cycle for Lane 1, Lane 2, Lane 3 and Lane 4 at scenario 1

Simulation results and corresponding graphs are given in the Table 2, Fig.6 and Fig.7 for scenario 2 below:
From camera readings: $N_1(1) = 19$, $N_2(1) = 10$, $N_3(1) = 08$, $N_4(1) = 23$

Table 2. Cycles and the optimum feasible results for scenario 2

Cycle k	Number of vehicles in lanes at the beginning of the cycle $N_j(k), j = 1,2,3,4$	Green signal time (sec.) $t_j(k), j = 1,2,3,4$	Cycle time (sec.) $C(k)$	Aggregate delay time at the end of the cycle (sec.) $D_j(k+1)$, $j = 1,2,3,4$	Number of vehicles in lanes at the end of the cycle $N_j(k+1)$, $j = 1,2,3,4$
1	19 10 08 23	10 10 07 19	50	788, 455, 377, 1175	17 10 07 19
2	17 10 07 19	13 10 06 16	49	617, 460, 340, 979	13 10 06 16
3	13 10 06 16	10 10 05 14	43	415, 387, 258, 724	10 09 06 13
4	10 09 06 13	08 08 05 11	36	264, 297, 210, 497	08 09 05 11
5	08 09 05 11	06 08 04 10	32	195, 255, 158, 373	07 08 05 09
6	07 08 05 09	06 07 04 08	29	146, 209, 142, 283	05 07 04 08
7	05 07 04 08	04 06 04 07	25	95, 155, 94, 215	04 07 03 07
8	04 07 03 07	03 06 03 06	22	69, 133, 64, 166	04 06 03 06
9	04 06 03 06	03 05 03 05	20	62, 106, 57, 131	04 06 03 06
10	04 06 03 06	03 05 03 05	20	62, 106, 57, 131	04 06 03 06

In Table 2 given above, the results of cycle 9 and cycle 10 are the same. If this process continues more cycles, the results will be same as the results of cycle 9. Because of oversaturation situation some vehicles are still waiting in each lane in the final cycle (cycle 9).
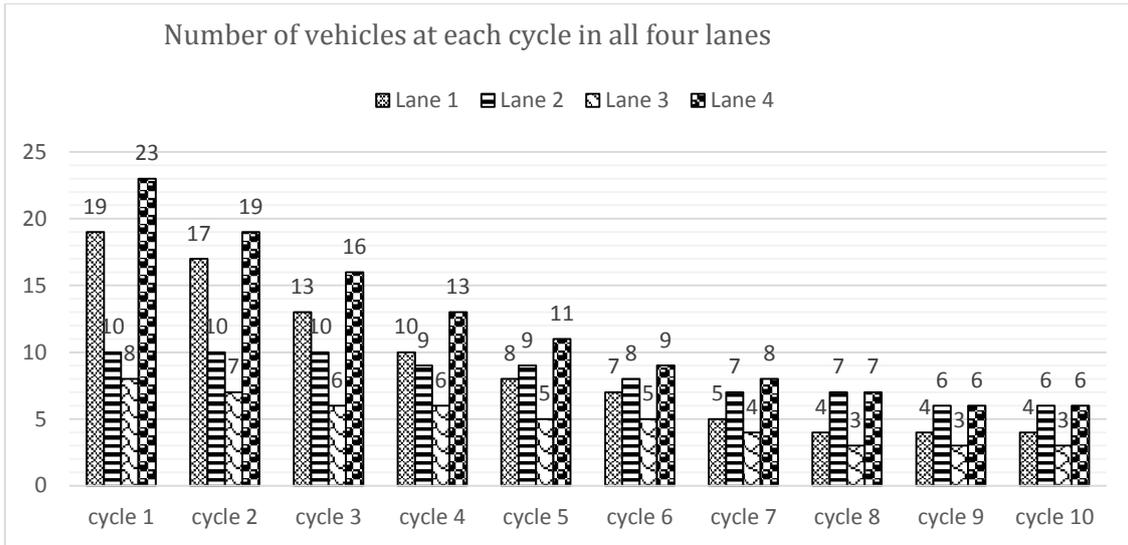
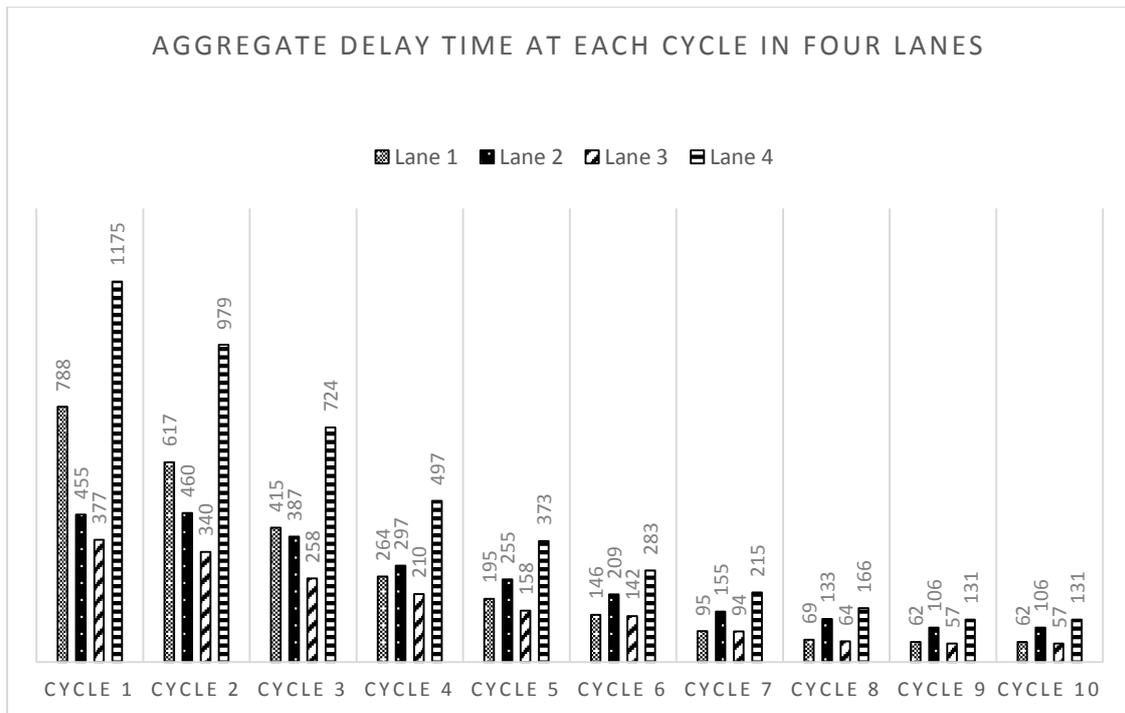**Fig.6.Number of vehicles at each cycle in Lane 1, Lane 2, Lane 3 and Lane 4 at scenario 2**

Fig.7. Aggregate delay time at each cycle for Lane 1, Lane 2, Lane 3, Lane 4 at scenario 2

V   CONCLUSION

A non-linear programming optimization model is developed to minimize the real time traffic signal control efficiency (number of vehicles and delay time) at a road intersection. A flow-chart is designed to solve the model with the aid of MATLAB optimization toolbox. Entering vehicles to the road intersection is counted using cameras installed in the road intersection which is proven to be an effective technique widely used around the world. This model incorporates inter green signal time. Our proposed model provides more practical solution than the existing developed models. The method is tested using hypothetical data set and considered two situations. Results are given in Table 1 and Table 2. It can be observed that the total number of vehicles waiting in each lane is decreasing from cycle to cycle. Subsequently, the cycle time and average aggregate delay time of vehicles are also decreasing. Finally, it can be concluded that the proposed model is delivering a promising result which can be practically implemented at road intersections consist of four lanes.

REFERENCES

[1]. Akcelik, R., Reilly, W. (eds), 1994.A Comparison of SCATS Adaptive and Fixed Time Control Techniques, Second International Symposium on Highway Capacity, Country reports, pp1932.

[2]. Tang-Hsien Chang, Jen-Ting Lin, August 2000."Optimal signal timing for an oversaturated intersection" Transportation Research Part B 34(2000)471-491.

[3]. AKCELIK. R. (1980). Time dependent Expressions for Delay, Stop Rate and Queue Length at Traffic signals. Internal Report. AIR 367-1. Australian Road Research Board,Vermount South, Australia.

[4]. Mrs. P.M.Daigavane, Dr. P.R.Bajaj, 2010. Real Time Vehicle Detection and Counting Method for Unsupervised Traffic Video on Highways, IJCSNS International Journal of Computer Science and Network Security, VOL.10 No.8.

[5]. AhmetYazici, GangdoSeo, UmitOzguner, A model predictive control approach for decentralized traffic signal control, Proc. 17th World Congress, The international federation of Automatic Control, Seoul, Korea, July 6-11,2008.