THE BARYON MASSES IN THE GROUND STATE [56,0+] AND EXCITED [56,2+] STATE USING $1/N_c$EXPANSION

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ABSTRACT:*This paper analyzes the masses of the light baryons in the ground state [56,0+] and excited [56,2+] state in the framework of the $1/N_c$ expansion using the available physical masses on Particle Data Group. All the calculations were done up to the leading order in SU(3) breaking and order $1/N_c$in $1/N_c$ expansion. The relevant operators to construct the mass formulas were identified and the mass formulas were developed for the ground state [56,0+] and excited [56,2+] state light baryons in $1/N_c$ expansion. The unknown dynamical coefficients of the mass formulas were fitted using the available empirical masses and lattice data sets. Using these coefficients, the theoretical masses of the light baryons in the ground state [56,0+] and excited [56,2+] state were calculated and found to be closely agreeing with the experimental masses. The masses of the 14 experimentally unfound baryons in the excited [56,2+] state were predicted.*
Keywords : Baryon masses, Particle Physics, $1/N_c$ expansion, Light Baryons, QCD

1. INTRODUCTION

Scientists paid their attention towards elementary particles since the earliest days and they have found that the fundamental constituents of matter are quarks and leptons. There are four fundamental forces of the universe and strong interaction is one of those whose force carrier is gluon. Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. The fundamental degrees of freedom of the QCD are the quarks and gluons and the fundamental parameters of QCD are the coupling constant and the quark masses.

QCD has two distinct regions namely, the asymptotic free region/ perturbative region and the confinement region/ non-pertubative region. In confinement region, the strong coupling constant which determines the strength of the interaction is large at low energies. Therefore, the low energy regime of QCD is not accessible to standard perturbative methods, because the corresponding running coupling constant becomes large at these scales.Thus, the strong coupling constant is not sufficient to be used as an expansion parameter in the low energy regime where ordinary baryons exist.The absence of a small expansion parameter at low and intermediate energy regimes makes it difficult to analyze the low energy properties of baryons.

‘tHooft proposed to generalize QCD to the arbitrary number of coloursN_c and realized that $1/N_c$is a hidden expansion parameter of QCD. $1/N_c$ expansion allowed a new perturbative approach in both high and low energy regimes. One of the main applications of $1/N_c$ expansion is to baryon masses.

1.1 The Standard Model

The theoretical framework, describing all the currently known elementary particles and their interactions, is known as the Standard Model. The particles in the Standard Model are in two basic types, leptons and quarks, each consisting of six particles. These quarks and leptons include every particle that is so far known. There are also four fundamental forces namely, the strong force, the weak force, the electromagnetic force and the gravitational force [13].

1.2 Quark

Quarks are half spin particles and there are six flavors of quarks namely up, down, charm, strange, top, bottom, categorized into three generations,

$$\binom{u}{d}$$

$$\binom{c}{s}$$

$$\binom{t}{b}$$

u and d quarks have the lowest masses of all quarks, and the heavier quarks decay through the weak interactions into u and d quarks. Thus, u and d quarks are the most stable and the most common in the universe. Each flavor of quark has its own mass and fractional electric charge. A quark also carries an additional quantum number called, colour, which takes one of the three values r, g or b.

1.3 Hadron

Quarks are not isolated particles and they are always found to have combined to form a colourless composite known as hadron. Hadrons are of two types namely, mesons and baryons. Bound states carrying non-vanishing baryon number are called baryons, while bound states carrying vanishing baryon number are called mesons.

1.4 Baryon

Hadrons composed of three quarks are termed to be baryons. Baryons are fermions which have half integer spin and they obey Fermi-Dirac statistics. They also possess fractional electric charge. Baryons that are commonly known are protons and neutrons that make most of the matter in the universe.

The 8 lightest baryons with spin 1/2 are arranged in a hexagonal array known as octet representation and the baryons with spin 3/2are arranged in a triangular array known as decuplet representation.

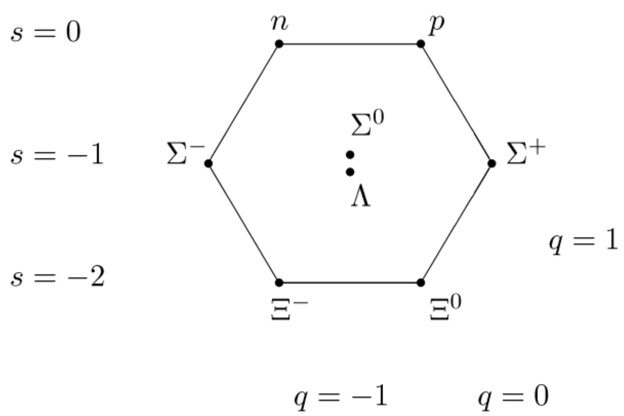

Figure. 1 Octet Representation of Baryons ($J^P = \frac{1}{2}^+$)

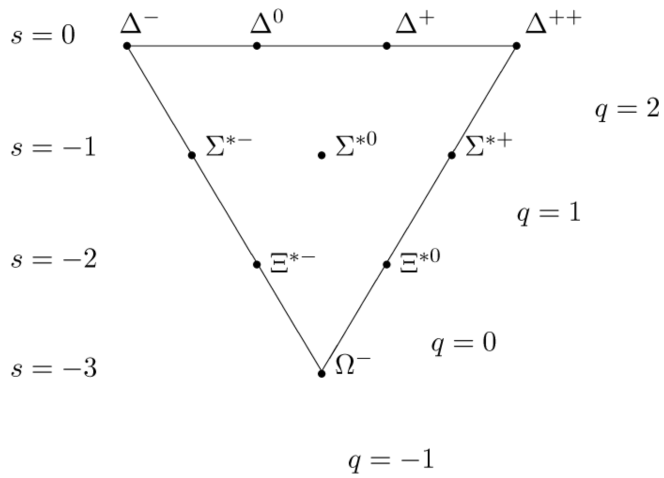

Figure. 2 Decuplet Representation Of Baryons ($J^p = \frac{3}{2}^+$)

2. METHODOLOGY

2.1 1/ N_c Expansion For Baryons

The strong coupling constant (α_s) which determines the strength of the strong interaction is one of the fundamental parameters of QCD that needs to be understood to understand the dynamical properties of QCD.

QCD has two distinct regions, asymptotic freedom region (at high energy) and confinement region (at low and intermediate energies). In asymptotic freedom region, the strong coupling constant is small and large in confinement region. The behavior of the strong coupling constant with energy is shown in figure. 3.

Since baryon masses are in the range of 1 or 2 GeV where the strong coupling constant (α_s) is very large, it is not suitable to be used for perturbation. Hence, a new expansion parameter $1/N_c$where $N_c$is the number of colours is used both in low and high energy regions of QCD.

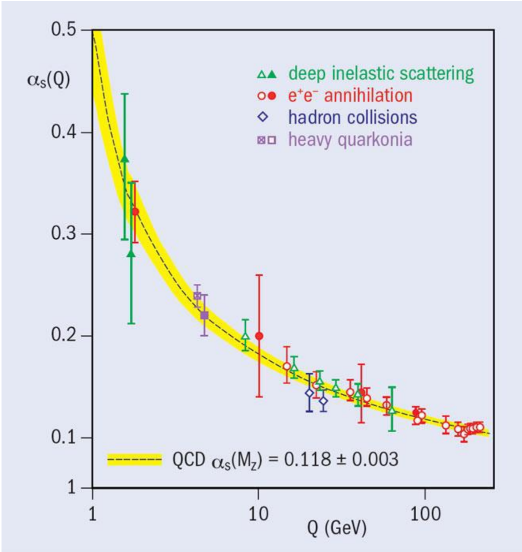

Figure.3 Behavior of strong coupling constant (α_s) as a function of the energy

2.2 The ground state [56,0+] masses in 1/$N_c$expansion

The effective operators that can be written in 1/$N_c$order are, i, J^2, T^2, G^2. The mass operator is expanded in terms of these effective operators,

$$M_{[56,0+]} = \sum m_i O_i$$

Where m_i is the unknown coefficient, in which all the complicated QCD dynamics is encoded.

Light quark baryon masses in 1/N_c expansion has 2 limits, the exact SU(3) symmetry limit in which u, d and s quark masses are considered to be the same and the SU(3) breaking limit in which u and d quarks are considered to have the same mass, but different from the s quark mass.

1) The exact SU(3) symmetry limit

   Baryon mass in exact SU(3) symmetry can be expanded in 1/N_c and given by,

$$M_B^{[0,0]} = \sum_m m_l^{(n)} \left(\frac{1}{N_c}\right)^{m-1} O_l^{(n)}$$

   Where $m_l^{(n)}$ are the unknown dynamical coefficients.

   For odd N_c,

$$M_B^{[0,0]} = m_0 N_c + m_1 J^2 \frac{1}{N_c}$$

2) The SU(3) breaking limit

   Baryon mass in the SU(3) breaking limit can be written as,

$$M_B^{[0,8]} = m_2 T^8 + m_3 J^i G^{i8} \frac{1}{N_c}$$

Therefore, the baryon mass equation up to O(1/N_c) in SU(3) breaking is given by,

$$M_B = m_0 N_c + m_1 J^2 \frac{1}{N_c} + m_2 T^8 + m_3 J^i G^{i8} \frac{1}{N_c}$$

Where, $M_B$is the baryon mass and m_0, m_1, m_2 and m_3 are unknown dynamical coefficients.

The operators T^8 and $J^i G^{i8}$ is given by[25],

$$T^8 = \frac{1}{\sqrt{12}}(N - N_s)$$

and

$$J^iG^{i8} = \frac{1}{2\sqrt{12}}(3I^2 - J^2 - 3J_s^2)$$

Where N_s is the number of strange quarks, I is the isospin, J is the total spin of the baryon and $J_s$is the spin of the strange quark. As the isospin effects are neglected, eight baryon states exist in ground state.

Table.1Matrix Elements of The Baryon Mass Operators

	N	**Λ**	**Σ**	**Ξ**	**Δ**	**Σ***	**Ξ***	**Ω**
$\mathbf{N_c}$	3	3	3	3	3	3	3	3
$\mathbf{N_s}$	0	1	1	2	0	1	2	3
$\mathbf{J^2}$	3/4	3/4	3/4	3/4	15/4	15/4	15/4	15/4
$\mathbf{J_s^2}$	0	3/4	3/4	2	0	3/4	2	15/4
$\mathbf{I^2}$	3/4	0	2	3/4	15/4	2	3/4	0
1	1	1	1	1	1	1	1	1
$\mathbf{J^2}$	3/4	3/4	3/4	3/4	15/4	15/4	15/4	15/4
$\mathbf{2\sqrt{3}T^8}$	3	0	0	-3	3	0	-3	-6
$\mathbf{4\sqrt{3}J^iG^{i8}}$	3/2	-3	3	-9/2	15/2	0	-15/2	-15

Since nucleon and delta particles are non-strange particles, the SU(3) breaking effect for them is zero. Therefore, T^8 and J^iG^{i8} are redefined as,

$$\overline{T^8} = T^8 - \frac{N_c}{\sqrt{12}}$$

and

$$\overline{J^iG^{i8}} = J^iG^{i8} - \frac{J^2}{\sqrt{12}}$$

Thus, the redefined matrix elements are,

Table.2Redefined Matrix Elements For Baryon Mass Operators

	N	**Λ**	**Σ**	**Ξ**	**Δ**	**Σ***	**Ξ***	**Ω**
1	1	1	1	1	1	1	1	1
$\mathbf{J^2}$	3/4	3/4	3/4	3/4	15/4	15/4	15/4	15/4
$\mathbf{2\sqrt{3}T^8}$	0	$-\sqrt{3}/2$	$-\sqrt{3}/2$	$-\sqrt{3}$	0	$-\sqrt{3}/2$	$-\sqrt{3}$	$-3\sqrt{3}/2$
$\mathbf{4\sqrt{3}J^iG^{i8}}$	0	$-3\sqrt{3}/8$	$\sqrt{3}/8$	$-\sqrt{3}/2$	0	$-5\sqrt{3}/8$	$-5\sqrt{3}/4$	$-15\sqrt{3}/8$

Hence, the mass equations for the ground state [56,0+] light baryons are,

$M_N = m_0N_c + \frac{3}{4}\frac{m1}{Nc}$

$M_\Sigma = m_0N_c + \frac{3}{4}\frac{m1}{Nc} - \frac{3}{\sqrt{12}}m_2 + \frac{3}{4\sqrt{12}}\frac{m3}{Nc}$

$M_\Lambda = m_0N_c + \frac{3}{4}\frac{m1}{Nc} - \frac{3}{\sqrt{12}}m_2 - \frac{9}{4\sqrt{12}}\frac{m3}{Nc}$

$M_{\Xi} = m_0 N_c + \frac{3}{4}\frac{m1}{Nc} - \frac{6}{\sqrt{12}} m_2 - \frac{3}{\sqrt{12}}\frac{m3}{Nc}$

$M_{\Delta} = m_0 N_c + \frac{15}{4}\frac{m1}{Nc}$

$M_{\Sigma^*} = m_0 N_c + \frac{15}{4}\frac{m1}{Nc} - \frac{3}{\sqrt{12}} m_2 - \frac{15}{4\sqrt{12}}\frac{m3}{Nc}$

$M_{\Xi^*} = m_0 N_c + \frac{15}{4}\frac{m1}{Nc} - \frac{6}{\sqrt{12}} m_2 - \frac{15}{2\sqrt{12}}\frac{m3}{Nc}$

$M_{\Omega} = m_0 N_c + \frac{15}{4}\frac{m1}{Nc} - \frac{6}{\sqrt{12}} m_2 - \frac{15}{2\sqrt{12}}\frac{m3}{Nc}$

2.3 The excited state [56,2+] masses in $1/N_c$expansion

The [56,2+] multiplet contains two flavor SU(3) octets with J = 3/2 , 5/2 and four decuplets with J = 1/2, 3/2, 5/2, 7/2. Altogether there are 24 baryons states out of which only 10 are experimentally established.

The mass operator can be expanded in $1/N_c$and given as,

$$M_{[56,2+]} = \sum c_i O_i + \sum b_j B_j$$

Where O_i are the SU(3) singlet operators and B_j are the SU(3) symmetry breaking operators. c_i and b_j are the effective coefficients. The operator basis contains three symmetric and three symmetry breaking operators at this order. The operators in the basis and the matrix elements are given in the table below,

Table.3Operator basis and matrix elements for the [56,2+] multiplet

	O_1 N_c 1	O_2 $1/_{Nc}\, l^i j^i$	O_3 $1/_{Nc}\, J^2$	B_1 - S	B_2 $1/_{Nc}\, l^i G^{i8} - 1/_{2\sqrt{3}}\, O_2$	B_3 $1/_{Nc}\, J^i G^{i8} - 1/_{2\sqrt{3}}\, O_3$
$N_{3/2}$	N_c	$-3/_{2Nc}$	$3/_{4Nc}$	0	0	0
$\Lambda_{3/2}$	N_c	$-3/_{2Nc}$	$3/_{4Nc}$	1	$3\sqrt{3}/_{4Nc}$	$-3\sqrt{3}/_{8Nc}$
$\Sigma_{3/2}$	N_c	$-3/_{2Nc}$	$3/_{4Nc}$	1	$-\sqrt{3}/_{4Nc}$	$\sqrt{3}/_{8Nc}$
$\Xi_{3/2}$	N_c	$-3/_{2Nc}$	$3/_{4Nc}$	2	$\sqrt{3}/_{Nc}$	$-\sqrt{3}/_{2Nc}$
$N_{5/2}$	N_c	$1/_{Nc}$	$3/_{4Nc}$	0	0	0
$\Lambda_{5/2}$	N_c	$1/_{Nc}$	$3/_{4Nc}$	1	$-\sqrt{3}/_{2Nc}$	$-3\sqrt{3}/_{8Nc}$
$\Sigma_{5/2}$	N_c	$1/_{Nc}$	$3/_{4Nc}$	1	$\sqrt{3}/_{6Nc}$	$\sqrt{3}/_{8Nc}$
$\Xi_{5/2}$	N_c	$1/_{Nc}$	$3/_{4Nc}$	2	$-2\sqrt{3}/_{3Nc}$	$-\sqrt{3}/_{2Nc}$
$\Delta_{1/2}$	N_c	$-9/_{2Nc}$	$15/_{4Nc}$	0	0	0
$\Sigma''_{1/2}$	N_c	$-9/_{2Nc}$	$15/_{4Nc}$	1	$3\sqrt{3}/_{4Nc}$	$-5\sqrt{3}/_{8Nc}$
$\Xi''_{1/2}$	N_c	$-9/_{2Nc}$	$15/_{4Nc}$	2	$3\sqrt{3}/_{2Nc}$	$-5\sqrt{3}/_{4Nc}$
$\Omega_{1/2}$	N_c	$-9/_{2Nc}$	$15/_{4Nc}$	3	$9\sqrt{3}/_{4Nc}$	$-15\sqrt{3}/_{8Nc}$
$\Delta_{3/2}$	N_c	$-3/_{Nc}$	$15/_{4Nc}$	0	0	0
$\Sigma''_{3/2}$	N_c	$-3/_{Nc}$	$15/_{4Nc}$	1	$\sqrt{3}/_{2Nc}$	$-5\sqrt{3}/_{8Nc}$

Ξ”$_{3/2}$	N_c	$-3/_{Nc}$	$15/_{4Nc}$	2	$\sqrt{3}/_{Nc}$	$-5\sqrt{3}/_{4Nc}$
Ω$_{3/2}$	N_c	$-3/_{Nc}$	$15/_{4Nc}$	3	$3\sqrt{3}/_{2Nc}$	$-15\sqrt{3}/_{8Nc}$
Δ$_{5/2}$	N_c	$-1/_{2Nc}$	$15/_{4Nc}$	0	0	0
Σ”$_{5/2}$	N_c	$-1/_{2Nc}$	$15/_{4Nc}$	1	$\sqrt{3}/_{12Nc}$	$-5\sqrt{3}/_{8Nc}$
Ξ”$_{5/2}$	N_c	$-1/_{2Nc}$	$15/_{4Nc}$	2	$\sqrt{3}/_{6Nc}$	$-5\sqrt{3}/_{4Nc}$
Ω$_{5/2}$	N_c	$-1/_{2Nc}$	$15/_{4Nc}$	3	$\sqrt{3}/_{4Nc}$	$-15\sqrt{3}/_{8Nc}$
Δ$_{7/2}$	N_c	$3/_{Nc}$	$15/_{4Nc}$	0	0	0
Σ”$_{7/2}$	N_c	$3/_{Nc}$	$15/_{4Nc}$	1	$21\sqrt{3}/_{8Nc}$	$-5\sqrt{3}/_{8Nc}$
Ξ”$_{7/2}$	N_c	$3/_{Nc}$	$15/_{4Nc}$	2	$21\sqrt{3}/_{4Nc}$	$-5\sqrt{3}/_{4Nc}$
Ω$_{7/2}$	N_c	$3/_{Nc}$	$15/_{4Nc}$	3	$63\sqrt{3}/_{8Nc}$	$-15\sqrt{3}/_{8Nc}$

2.4 Fitting unknown dynamical coefficients

The unknown dynamical coefficients in the theoretical mass expression for the light baryons in the ground state [56,0+] and excited [56,2+] state were fitted using empirically known baryon masses which are listed in the Particle Data Group. The best fit is obtained by minimizing the chi-square value with respect to the dynamical coefficients. This was done using Mathematica software.

$$\chi^2 = \sum \left(\frac{M_{Theoretical}[i] - M_{Experimental}[i]}{M_{Error}[i]}\right)^2$$

Where,

$M_{Theoretical}$ - Theoretical baryon masses

$M_{Experimental}$ - Experimental baryon masses

M_{Error} - Error associated with the experimental masses

3. DISCUSSION AND RESULTS

The values of the unknown dynamical coefficients for the light baryons in the ground state [56,0+] and excited [56,2+] state obtained are listed in the tables below along with their calculated errors.

Table.4Theoretical values of dynamical coefficients in the mass equations of ground state [56,0+] light baryons using PDG fit

Dynamical coefficient	Value (MeV/c^2)
m_0	$286.5456628 \pm 0.0117185$
m_1	317.1270359 ± 0.008775

m_2	$-270.4222148 \pm 0.0278867$
m_3	265.310925 ± 0.0023524

Table.5Theoretical values of dynamical coefficients in the mass equations of ground state [56,2+] light baryons using PDG fit

Dynamical coefficient	Value (MeV/c^2)
c_0	$539.5887552 \pm 0.0124968$
c_1	14.2231518 ± 0.0005445
c_2	$247.4445486 \pm 0.0227168$
b_0	$211.5498827 \pm 0.0019342$
b_1	7.2701789 ± 0.0008665
b_2	$341.7137237 \pm 0.0024677$

The theoretical masses of the 8 light baryons in the ground state [56,0+] and the 24 light baryons in the excited state [56,2+] were calculated by providing the calculated theoretical values for the unknown dynamical coefficients to the theoretical baryon mass expressions. This includes the experimentally unfound masses of 14 excited state [56,2+] baryons. The comparison between the theoretical and experimental baryon masses of the ground state [56,0+] and excited [56,2+] state respectively are tabulated below.

Table.6Comparison of the theoretical and experimental (PDG values) baryon masses of the ground state [56,0+]

Baryon State	Experimental Value (MeV/c^2)	Theoretical Value (MeV/c^2)
N	$938.9187473 \pm 0.0000082$	$938.9187473 \pm 0.0329618$
Λ	1115.683 ± 0.006	$1115.6697549 \pm 0.0083019$
Σ	$1193.153667 \pm 0.0761955$	$1192.2584218 \pm 0.0089810$
Ξ	1318.285 ± 0.211896	$1330.7150959 \pm 0.0160184$
Δ	1232 ± 2	$1256.0457832 \pm 0.0241867$
Σ*	1384.566667 ± 1.17154	$1394.5024572 \pm 0.0008127$
Ξ*	1533.4 ± 0.68	$1532.9591312 \pm 0.0258121$
Ω	1672.45 ± 0.29	$1671.4158053 \pm 0.0508115$

Table.7Comparison of the theoretical and experimental (PDG values) baryon masses of the excited state [56,2+]

Baryon State	Experimental Value (MeV/c^2)	Theoretical Value (MeV/c^2)
$\mathbf{N_{3/2}}$	1700 ± 50	$1673.5158269 \pm 0.0315389$
$\mathbf{\Lambda_{3/2}}$	1800 ± 30	$1814.2305980 \pm 0.0325635$
$\mathbf{\Sigma_{3/2}}$	---	$1908.6774134 \pm 0.0337763$
$\mathbf{\Xi_{3/2}}$	---	$2002.1687769 \pm 0.0341946$
$\mathbf{N_{5/2}}$	1683 ± 8	$1685.3684534 \pm 0.3199264$
$\mathbf{\Lambda_{5/2}}$	1820 ± 5	$1820.8364248 \pm 0.0336427$
$\mathbf{\Sigma_{5/2}}$	1918 ± 18	$1922.2789731 \pm 0.0340216$
$\mathbf{\Xi_{5/2}}$	---	$2007.0256704 \pm 0.0354822$
$\mathbf{\Delta_{1/2}}$	1895 ± 25	$1906.7372238 \pm 0.0082776$
$\mathbf{\Sigma''_{1/2}}$	---	$1998.1298674 \pm 0.0089461$
$\mathbf{\Xi''_{1/2}}$	---	$2089.5225109 \pm 0.0096146$
$\mathbf{\Omega_{1/2}}$	---	$2180.9151544 \pm 0.0102831$
$\mathbf{\Delta_{3/2}}$	1935 ± 35	$1913.8487997 \pm 0.0085498$
$\mathbf{\Sigma''_{3/2}}$	---	$2004.1920833 \pm 0.0093434$
$\mathbf{\Xi''_{3/2}}$	---	$2094.5353669 \pm 0.0101370$
$\mathbf{\Omega_{3/2}}$	---	$2184.8786505 \pm 0.0109306$
$\mathbf{\Delta_{5/2}}$	1895 ± 25	$1925.70142613 \pm 0.0090036$

Σ”$_{5/2}$	---	2014.2957765 $\pm$ 0.0100056
Ξ”$_{5/2}$	---	2102.8901269 $\pm$ 0.0110077
Ω$_{5/2}$	---	2191.4844773 $\pm$ 0.0120098
Δ$_{7/2}$	1950 $\pm$10	1942.2951032 $\pm$ 0.0096388
Σ”$_{7/2}$	2033 $\pm$8	2041.5579463 $\pm$ 0.0093693
Ξ”$_{7/2}$	---	2140.8207894 $\pm$ 0.0090998
Ω$_{7/2}$	---	2240.0836325 $\pm$ 0.0088303

4. CONCLUSION

$1/N_c$ expansion is an effective tool to understand and analyze baryons that lie in the confinement region. The unknown baryon masses can be determined and predicted by constructing baryon mass expressions with unknown dynamical coefficients. This was employed to the ground state [56,0+] baryons and was found that the theoretically calculated masses closely tallied with the experimentally found masses. The same was applied to the excited state [56,2+] baryons and was found that the theoretically calculated masses closely agreed with the experimental values of the known baryon masses. The unknown masses of 14 baryons in the excited state [56,2+] were predicted theoretically.

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