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DOES LKR/AUD EXCHANGE RATE EXHIBIT LONG MEMORY? A FRACTIONAL INTEGRATION APPROACH

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Abstract

This study employs the new science of 'econophysics' approach to explain the behavior of exchange rate between Sri Lankan rupees and Australian dollar (LKR/AUD). The study covers the period from January 1, 1990 to December 12, 2017, consisting of 7212 observations. The return of the LKR/AUD is defined as $\left[r_t = [\ln(ER_t) - \ln(ER_{t-1})]*100\right]$. ARFIMA(p,d,q)) model and FIGARCH(p,d,q) model are used to examine the presence of fractional integration in the return series. The time domain exact maximum likelihood is used to estimate the ARFIMA model. Volatility of LKRAUD exchange rate return series are proxied by absolute return, squared return and conditional variance derived from FIGARCH model. The autocorrelation of all proxies decay hyperbolically for lags 1 through 200. The results show that return series does not exhibit long memory while conditional variance of return series, absolute return series and squared return series do. The estimate of Long memory parameter '*d*' for the return series indicates that return series seems to have short memory. However, the squared return series, conditional variance series and absolute return series exhibit long memory as the statistic of memory parameter '*d*' are statistically significant at 1% significance level and lie within the interval 0 to 0.5. Visual inspection and inferential results reveal strong evidence of long memory property in the volatility of daily LKR/AUD exchange rate return. Shocks to the exchange rate persist over a long period of time. The findings indicate that these markets are not efficient. Hence, the results provide information to the investors, traders and government policy makers to add some risk in their strategies.

Keywords: ARFIMA, exchange rate, FIGARCH, fractional integration, Long memory, Sri Lanka

1. Introduction

In today's complex and interdependent business world, exchange rate movements have a significant impact on the world's political and economic stability as well as on the welfare of individual countries. Basic understanding of the statistical properties of stochastic behavior of exchange rates is important for policy makers and investors and financial analysists. It will assist investors to make appropriate investment decisions. Statistical characteristics are extremely helpful for researchers who seek to understand exchange rate changes statistically and simulate them efficiently. The question of whether exchange rate markets are efficient or not, is directly related to the long memory (LM) in the exchange rate changes. Therefore, detecting LM in an exchange rate dynamics is important to understand whether exchange rate markets of an economy are efficient or not. LM would suggest a very strong market inefficiency. It is extremely important to understand not only return of exchange rates but also volatility of exchange rate return in an exchange rate markets.

The ARIMA model examines the temporal dynamics of an economic variable as *integer* integration, under which time series are presumed to be integrated to order *zero* or *one.* In addition, Conventional unit root tests account only integer values. This is highly restrictive, being constrained to the integer domain. For example, in practice, some series do not possess a unit root while they show signs of dependence. Recent advances in computing and econometric techniques have motivated researchers to consider the fractional values of integration of the time series. The concept of fractional integration reveals the hidden characteristics of the LM and the short memory (SM) of economic stationary time series. Fractional differencing parameter approach accounts the fractional values so that it generalizes ARIMA model and offer more flexibility in modeling to explain both short term, long term correlation structure of a time series. However, there are scarce studies investigating the memory properties of Sri Lankan exchange rates using fractional integration technique.

This is a new attempt to study the LKR/AUD exchange rate dynamics based on fractional integration approach in Sri Lanka. Many studies have attempted to study the exchange rate dynamics. However, their focus has been mainly on economic aspects, such as factors affecting it, or impact of exchange rate changes on the economy. There exists no in-depth scientific technical analysis on LM of volatility dynamics of the exchange rate changes. This study intends to fill this gap in the finance literature and provide an in-depth analysis. A comprehensive understanding of time series and statistical properties of LKR/AUD exchange rate in Sri Lanka might provide useful implications for the direction of future research and effective exchange rate and monetary, and trade policies. Results of this study may provide relevant implications for investors and policy makers. Therefore, this study would contribute significantly to the existing knowledge. The aim of this study is to examine for the LM property in the LKR/AUD daily exchange rate using fractional dynamics approach. The specific aims are to estimate the fractional integration parameter for first and second moment of LKR/AUD exchange rate return series using ARFIMA and fractionally integrated GARCH (FIGARCH) models.

2. Review of Literature

Theoretical studies

Autoregressive fractionally integrated moving average model

Granger and Joyeux (1980) and Hosking (1981) introduced the ARFIMA model which is now widely used in practice. This is a parsimonious and flexible model to study long memory and short run dynamics simultaneously. Fractional integration is a more general way to describe long-range dependence than integer integration specification. ARFIMA(*p,d,q*) process can be written as : $\phi(L)(1-L)^d r_t = \theta(L)\varepsilon_t$ where *d* is a fractional differencing parameter, L is the lag operator, $\phi(L)$, $\theta(L)$ are lag polynomials of finite orders. $\phi(L) = 1-\phi L - \phi_2 L^2 - .... - \phi_p L^p$ is the autoregressive polynomial finite orders, $\theta(L) = 1+\theta_1 L + \theta_2 L^2 + ... + \theta_q L^q$ represents the moving average polynomial of finite orders, the roots of $\phi(L)$, and $\theta(L)$ lie outside the unit circle , and $\varepsilon_t \sim iid(0,\sigma^2)$, and $(1-L)^d$ is the fractional differencing operator defined as an infinite binomial series expansion in powers of the lag operator.

$$[1] \quad (1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-L)^j = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)}$$

where $\Gamma(.)$ is the gamma function, r_t is both stationary and invertible if the roots $\Phi(L)$ and $\Theta(L)$ are outside the unit circle, and $d < |0.5|$. The parameter *d* is allowed to assume any real value. This model permits the degree of differencing (*d*) to take fractional values. LM processes are stationary processes whose autocorrelation functions decay more slowly than SM processes. Hosking (1981) showed that the autocorrelation, $\rho(k)$ of an ARFIMA processes is proportional to k^{2d-1} as $k \to \infty$, [$\rho(k) \propto k^{2d-1}$]. Note k=lags. It implies that the autocorrelations of ARFIMA processes decay hyperbolically to zero as $k \to \infty$. In the time domain, a hyperbolically decaying auto-correlation function characterizes the presence of LM. Thus, ARFIMA (*p, d, q*) processes are known to be capable of modeling long-run persistence. The *d* is the fractional integration parameter that measures long range dependence of the series. The power spectrum of the ARFIMA (p,d,q) process Yt is given by

[2] $$f(\lambda) = \{4\sin^2(\lambda/2)\}^{-\delta} f_e(\lambda), \qquad \lambda \in [0,\pi]$$

where $f_e(\lambda)$ is the power spectrum of the ARMA (p,q) process that is positive and bounded. In case of long memory, $\delta > 0$, the spectrum of Yt is unbounded for frequencies approaching 0; $\lim_{\lambda \to 0} f(\lambda)\lambda^{2\delta} = f_e(0)$

Fractionally integrated GARCH model

Baillie, Bollerslev, and Mikkelsen (1996) introduced the fractionally integrated GARCH (FIGARCH) model to describe the volatility dynamics. In the FIGARCH model, the persistent behavior of volatility is modelled using a fractional difference parameter (*d*), while short term volatility is modeled by conventional ARCH and GARCH parameters. GARCH model is stated as

[3] $$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

where,

$$\beta(L) = 1 - \beta_1 L - \beta_2 L^2 - .... - \beta_p L^p, \quad \alpha(\mathrm{L}) = 1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_p L^p$$

The roots of the polynomials of $\alpha(L), \beta(L)$ lie outside the unit circle.

In addition, the LM parameter for series LKR/AUD return volatility series is estimated by using the fractional integration GARCH (FIGARCH) model for variance equation. FIGARCH model is stated as

[4] $$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]\nu_t$$

where 0<d<1, and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle . The value of the fractional differencing parameter depends on the decay rate of a shock

Empirical studies

Corazza and Malliaris (2002) using Hurst exponent carry out a study on foreign currency markets and find evidence of long memory. Nath and Reddy (2002) employed Hurst R/S statistics and a variance ratio test to analyze the effect of long memory on the Indian foreign exchange market against the US dollar. Kyungsik Kim *et al.*, (2012) investigate the multifractal properties of three foreign exchange rates Korean won, Japanese yen and Euro against the US dollar that are quoted with different economic scales using generalized Hurst exponent and the autocorrelation function.

Peters (1991) study the LM characteristics of daily exchange rate data of USD, Japanese yen, GBP, Euros and Singapore dollar and found evidence of LM for using R/S approach. Bhar (1994) examines LM in the Yen/dollar exchange rate and found no evidence of LM indicating efficient pricing by the market participants for using modified R/S approach.

Sasikumar (2011) analyzed the presence of LM in Indian foreign exchange market using Hurst exponent, Hurst-Mandelbrot R/S statistic, Lo's modified R/S statistic, Robinson's semi parametric estimator and Andrew- Guggenburger modified GPH estimator. The results showed that the Indian foreign exchange market exhibits LM. Sen et al. (2010) employed the R/S statistic, modified R/S statistic, Whittle test, and Hurst exponent to test the presence of LM in nine selected currencies around the globe in terms of the Indian Rupee and found significant presence of LM in appreciation and/or depreciation in these nine exchange rates.

Christos Floros (2008) test for the presence of fractional integration, or LM, in the daily returns of 34 exchange rates against the US dollar using ARFIMA (p,d,q) models. Alpana Vats (2011) study the long memory behavior of daily returns of Chinese Yuan, Indonesian Rupiah and Taiwan Dollar using fractionally integrated models, ARFIMA and FIGARCH to investigate the LM property in returns and volatility respectively.

Cheung (1993) first studied time series properties of five major nominal exchange rates series using ARFIMA model that provides a direct and convenient frame work to study both short and LM behavior.

However, there are no studies investigating the memory properties of Sri Lankan exchange rates using fractional integration technique. Therefore, this study will be a new attempt to study about LKR/AUD exchange rate dynamics based on recent econometric time series analysis in Sri Lanka.

3. Methodology

Data

Data used in this study are daily exchange rate of Sri Lankan rupee against Australian dollar, indicated by LKR/AUD. The study cover the period from January 1, 1990 to December 12, 2017, consisting of 7212 observations. These data were collected from the Datastream (Datastream, 1995). The daily changes of the exchange rate are measured by the return series defined as

[1] $$r_t = [\ln(ER_t) - \ln(ER_{t-1})]*100$$

where ER_t=LKR/AUD is nominal exchange rate. LLKRAUD=ln(LKR/AUD). The absolute return ($|r_t|$) and squared return (r_t^2), conditional variance derived from FIGARCH model are used as proxies for the volatility of LKR/AUD.

Analytical tools

Standard unit root tests were employed to check existence of unit root on the return series.

Unit root tests

Before estimating the LM parameter, we tested whether the LKR/AUD return is non-stationary using standard unit root tests; the Augmented Dickey-Fuller (ADF) test, the Phillips and Perron (PP) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are used to test the stationarity of the series. These tests can only identify whether series are I(0) or I(1).

Autoregressive Fractionally Integrated Moving Average Model:

This study employs ARFIMA(*p,d,q*) frame work to test LM in LKR/AUD exchange rate dynamics. The ARFIMA model is used to estimate the parameter d that describes the long-term memory of a series of conditional mean. If the parameter d is statistically significant, it indicates the evidence of LM. Using the theoretical knowledge given above, we test whether the return of LKR/AUD exchange rate has LM. When d=0, the series is stationary, and shows SM and mean reversion with finite variance. In this case, the effects of a shock in a variable are transitory. When d=0, ACF decays exponentially to zero. If d=0.5, the process is invertible but nonstationary. If d= -0.5, the process is stationary but not invertible. When d=1, the series is integrated order one, having unit root, being non-stationary, with infinite variance, and is non-mean reverting. In this case, the effect of a shock in the series is permanent, having a long-term effect, forever persistent. If d>1, the series is non-stationary, non-mean reverting, with infinite memory. In this case, the effect of a shock is permanent divergence. For d<1, the series is mean-reverting. When $d \geq 0.5$, the series does not have stationary covariance, and consequently it has infinite covariance (Baillie et al. 1996). Therefore the process is non-stationary (Granger and Joyeux, 1980); however, long-range dependence is associated with all nonzero, d >0. Thus, the memory property of a process depends significantly on the value of d. For $-0.5 < d < 0$, the series is stationary, with intermediate memory and anti-persistent. If $0 < d < 0.5$, the series has long memory, and behaves as if fractionally integrated, indicating strong dependence between past observations, LM, mean reversion, and covariance stationary. The autocorrelations are positive and the ACF decay hyperbolically and monotonically towards zero as the lag length increases. The correlation between distant observations can be relatively high, implying that LM exists. The effects of a shock in real output last in the long run. When $0.5 < d < 1$, the process still has LM, but the series is no longer covariance stationary and mean reverting. The effect of a shock in the series is long-lasting and decays at an even slower rate. Thus, the memory property of a process depends significantly on the value of d.

FIGARCH Model

LKR/AUD return volatility series is estimated by using the fractional integration GARCH (FIGARCH) model for variance equation. FIGARCH model is stated as

[6] $$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]v_t$$

where 0<d<1, and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle . The value of the fractional differencing parameter is estimated for the return of LKRAUD exchange rate series.

4. Results and Discussions

This section describes the empirical results of ARFIMA and FIGARCH models application for LKRAUD exchange rate. ARFIMA model is estimated to examine long memory of LKRAUD return series and FIGARCH model is estimated to examine long memory of volatility of (second moment) LKRAUD exchange rate return series.

4.1 Basic features of LKR/AUD exchange rate dynamics in Sri Lanka

Figure 1 exhibits a time series plot of LKR/AUD and return of LKR/AUD. LKR/AUD series are moving upward with volatile. Visual inspection indicates that the series seems to be non-stationary. The first and second moment of the LKR/AUD distribution vary over time. Return series appear to be random fluctuations around zero and with time varying variance. However, the return series behave differently. Return series shows that the first moment seems to be constant over time, but the second moment of the return vary over time. There exists a volatility clustering salient feature in the LKR/AUD return dynamics.

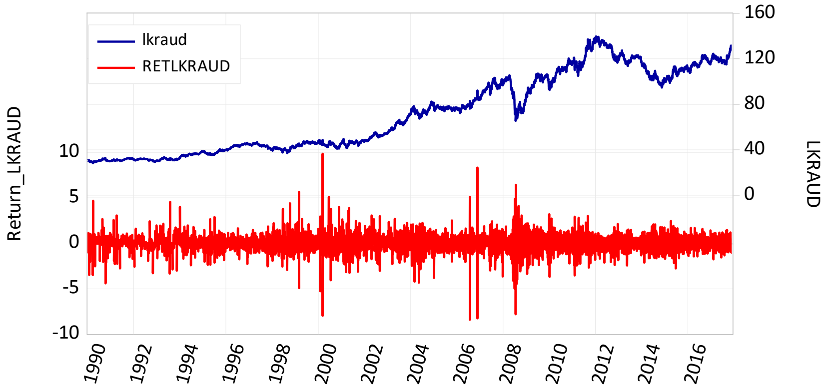

Figure 1 LKR/AUD and its return behavior, January 3, 1990 to December 12, 2017

Further, Figure 1 shows that return series process has mean zero with non-constant variances conditional on the past. However, when we extract the trend using Hodrick-Prescott (HP) filter , the trend of the return series is not linear and having volatile trend. (Figure 2)

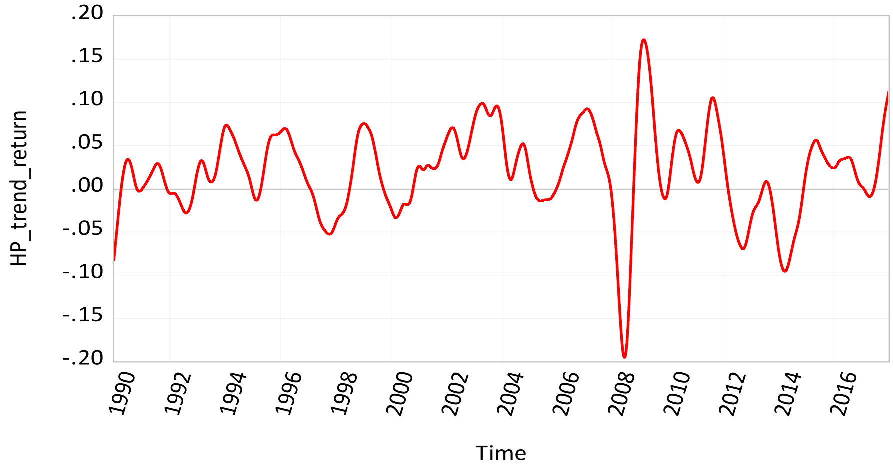

Figure 2 HP trend of the return series

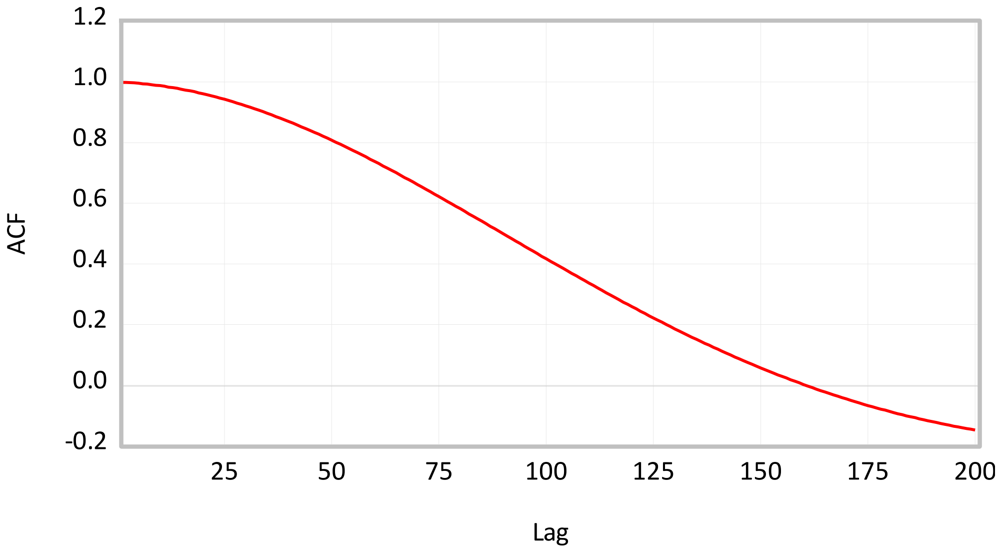

Figure 3 ACF of HP trend for returns of LKRAUD for lags 1 to 200

Figure 3 the autocorrelation of LKR/AUD return filtered by HP method decay slowly, at hyperbolic rate. This figure provides strong evidence of LM. That is the impact of shock ε_t on LKR/AUD return does not diminish over time.
The Figures 4, Figure 5, and Figure 6 shows the dynamic pattern of the volatility proxies of LKRAUD return.

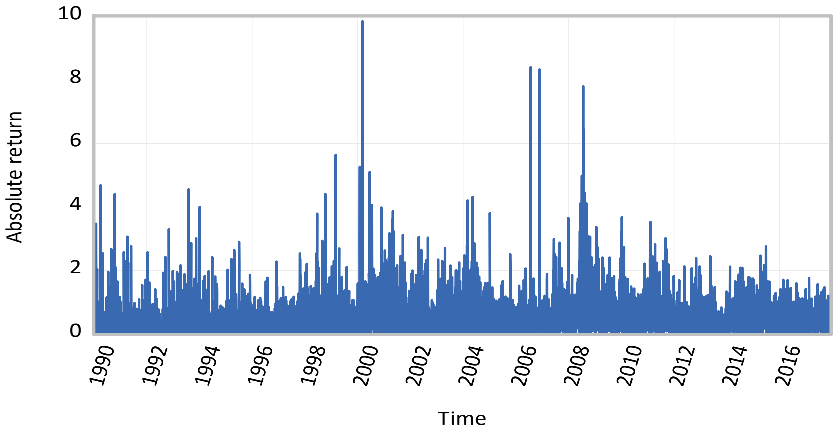

Figure 4 Absolute return dynamics

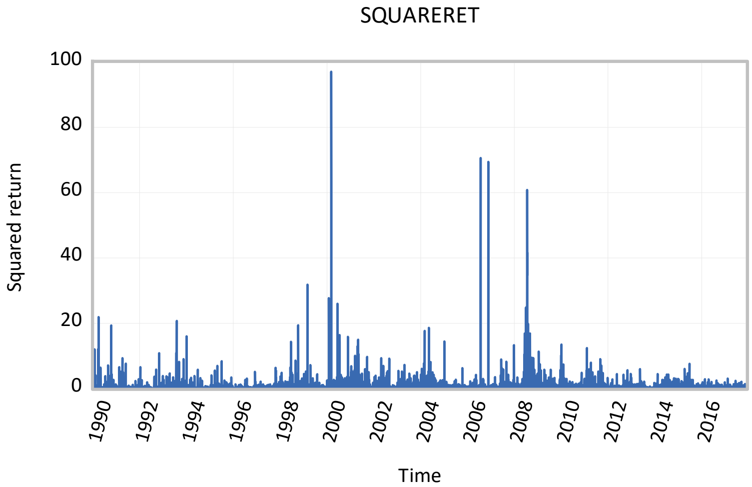

Figure 5 Squared return dynamics

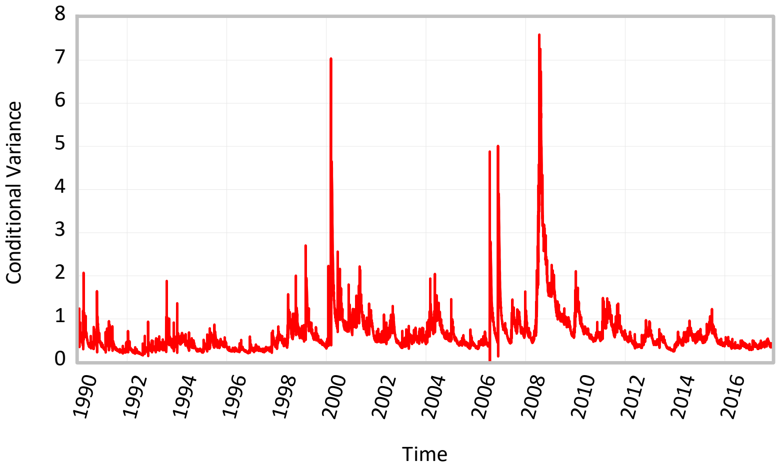

Figure 6 Conditional variance dynamics

The ACF of all proxies decay hyperbolic rate indicating LM

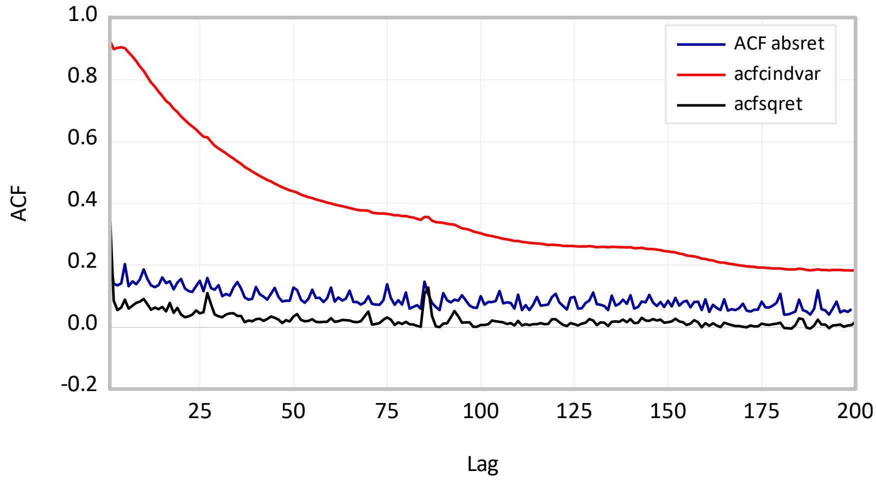

Figure 7 ACF of squared return, absolute return and conditional variance of LKRAUD for lags 1-200.

Unit root test results

The results of unit root tests given in the Table 1 indicate that Exchange rate at level is nonstationary.

Table 1: Unit root test results

Exchange Rate	**Level with intercept**		
	ADF	**PP**	**KPSS**
LLKRAUD (Level)	-1.472	-1.497	4.661
	(0.547)	(0.535)	[0.463]
RETURN of LKRAUD	-62.400	-65.358	0.057
	(0.000)	(0.000)	[0.463]
ABSRETURN of LKRAUD	-9.563	-61.860	**0.8544**
	(0.000)	(0.000)	[0.463]
SQRETURN of LKRAUD	-12.318	-44.900	**0.832**
	(0.000)	(0.000)	[0.463]

(P values are given in parenthesis. Critical value (5%) is given in square bracket)

The return series is stationary as the p vale is less than 0.05 for ADF, PP and KPSS tests. However, in the case of volatility proxies, KPSS test contradictory to ADF and PP test results shows that volatility proxies; Absolute, squared returns series are not stationary series. They seems to be nonstationary.

Estimates of ARFIMA model

The estimated results of ARFIMA model using STATA are presented in Table 2.. Return series seems to have short memory as d is less than 0. It indicates anti persistent. LL indicates the log likelihood ratio, p values are in parentheses, d indicates fractional difference parameter. All AR, MA parameters are significantly different from zero.

Table 2: Results from ARFIMA (1,*d*,1) exchange rate return series

Items	**constant**	***d***	**LL**	**Wald Chi²**
Return LKRAUD	**0.019** **(0.000)**	**-0.051** **(0.000)**	**-8769.32**	**34.0** **(0.000)**

Note: Probability values are in parenthesis

Volatility Model: FIGARCH model estimates

Though return series is stationary and has short memory, volatility proxies are not stationarity. Unit root test (KPSS), fractional difference parameter estimates shows that volatility proxies; conditional variance, squared return are nonstationary and have long memory. The estimates of FIGARCH model shows that d is significantly different from zero. Long memory parameter estimates for volatility proxies are statistically significant different from zero. It indicates that volatility series of LKRAUD return exhibit long memory. Long memory volatility series tends to change quite slowly over time. The effects of a shock takes a considerable time to decay.

Table 3 FIGARCH (*1,d,1*) model was fitted for return of LKRAUD exchange rate

Variables	Constant	*d*	ARCH	GARCH	LL
Squared return	0.879 (0.190)	0.794 (0.000)	0.196 (0.000)	0.579 (0.000)	49501.30
Absolute return	0.701 (0.131)	0.419 (0003)	0.527 (0.000)	0.797 (0.000)	-6187.66
Conditional Variance	-0.002 (0.250)	0.748 (0.0006)	0.362 (0.000)	0.815 (0.000)	-5806.68

(P values are in parenthesis)

5. Conclusions

This study has examined the long memory property of LKR/AUD exchange rate for the period from January 1, 1990 to December 12, 2017. ARFIMA models are estimated using maximum likelihood method. Results suggests that LKR/AUD return exhibit short memory while volatility proxies; absolute, squared return and conditional variance shows long memory and nonstationary. LM in the first moment of the return is relatively weaker than that in the second moment of the return. This indicates that shocks to the exchange rate persist over a long period of time. This also indicates that exchange rate markets are not efficient, not stable. The findings have important policy implications for government policy makers and participants of the foreign exchange market of Sri Lanka. Hence, results provides information to the investors and traders to add some risk to their strategies. Hence, these findings are useful to the traders, investors in the foreign exchange markets.

References

Alpana Vats (2011). Long Memory in Return and Volatility: Evidence from Foreign Exchange Market of Asian Countries. The International Journal of Applied Economics and Finance, 5(4), 245-256

Bhar, R. (1994). Testing for long-term memory in Yen/Dollar Exchange Rate. Financial Engineering and the Japanese Markets 1, 101-109

Corazza, M. and Malliaris, A.G. (2002). Multifractality in Foreign Currency Markets, Multinational Finance Journal, 6, 387-401

Christos Floros (2008). Long Memory in Exchange Rates: International Evidence. The International Journal of Business and Finance Research, Volume 2, Number 1, 31-39

Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long memory time series models. Journal of time series analysis, 4(4), 221-238.

Granger, C. W. and Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. Journal of time series analysis, 1(1), 15-29.

Hosking, J. R. (1981). Fractional differencing. Biometrika, 68(1), 165-176. http://doi: 10.1093/biomet/68.1.165.

Hurst, H. E. (1951). Long-term storage capacity of reservoirs. Trans. Amer. Soc. Civil Eng., 116, 770-808.

Kyungsik Kim, Jae-Won Jung, Sehyun Kim and Soo Yong Kim. (2012). Dynamical Analyses of the Time Series for Three Foreign Exchange Rates. Journal of the Korean Physical Society, Vol. 60, No. 9, pp. 1473-1476

Mandelbrot, B. B. and Wallis, J. R. (1969). Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence. Water Resources Research, 5(5), 967-988.

Nath, G.C. and Reddy, Y. V. (2002). Long Memory in Rupee-Dollar Exchange Rate– An Empirical Study, UTIICM Conference Paper.

Peters, E. (1991). Chaos and order in the capital markets: A new view of cycles, prices, and market volatility. New York: John Wiley & Sons.

Sasikumar, A. (2011). An analysis of long memory in the Indian foreign exchange market. GITAM review of International Business, 4(1), pp.23- 34.

Sen, C., Chakrabarti, G. and Sarkar, A. (2010). Long Memory in Foreign Exchange Market: Mythr Reality? A study of selected

Sivarajasingham, S. and Balamurali, N. (2017). Testing for long memory in the LKR/USD exchange rate: evidence from Sri Lanka. Journal of Business Studies, 4(1)