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Categorical Properties of Mackey Functors

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Abstract: We develop the categorical properties of Mackey functors. Mackey functors became important in the theory of representation of groups since groups have been studied using Mackey functors during the last 40 years. In this paper we use the categorical definition for Mackey functors and investigate their categorical properties on compact Lie groups. We show that the category of Mackey functors on a compact Lie groups is a monoidal category and its monoids are Green functors.

Keywords: Monoidal category, Monoids, Abelian categories.

Introduction

Macky functors is an algebraic structure which behave like the induction, restriction and conjugation mappings in group representation theory. Mackey functors were first introduced by J.A. Green and A. Dress in the early 1970's as a tool for studying representations of finite groups and their subgroups. There are (at least) three equivalent definitions of Mackey functors for a finite group *G* (Bouc, 1997). The first one defines the Mackey functors as a poset of subgroups of a group *G* (Green,1971). This is the most accessible definition of a Mackey functor for a finite group is expressed in terms of axiomatic relations which were developed by A. Green. The second one defines the Mackey functors in the sense of categorical way which was defined by A. dress (Dress,1973). The third one defines the Mackey functors as modules over the Mackey algebra (Thevenaz and Webb, 1995). But they all amount to the same thing. In this paper we use the second definition which defines the Mackey functors in terms of categories.

Many of the fundamental results on Mackey functors for a finite group are extended to Mackey functors for a compact Lie group. Mackey functors have been studies on finite groups for a long time. The study of Makey functors for an infinite group has appeared recently. There is also a new concept called globally-defined Mackey functors which were appeared more recently.

Some examples of Mackey functors for finite groups are representations rings, Burnside rings, group cohomology, equivariant cohomology, equivariant topological K-theory, algebraic K-theory of group rings and algebraic K-theory. One application of Mackey functors to number theory has been to provide relations between $\lambda-$ and $\mu-$ invariants in Iwasawa theory and between Mordell-Weil groups, Shafarevich-Tate groups and zeta functions of elliptic curves.

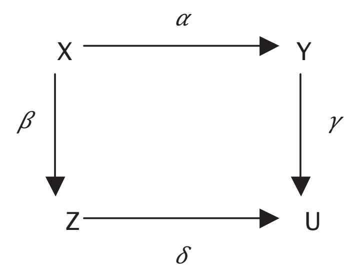
Methods

Mackey functors were defined by at least three equivalent definitions. We use the definition developed by A. Dress which describes the Mackey functors in categorical way. The important of this definition is that it has less dependent on a large number of axioms. This is defined using the category of G-sets whose objects are the finite let G-sets, and whose morphisms are the G-equivariant mappings.

1. Definition of Mackey Functors

A Mackey functor over a ring R is a pair of functors M=(M_*, M^*) from the category of G-sets to the category of R-modules, R-Mod, so that M_* is covariant and M^* is contravarient, $M_*(X)$=$M^*(X)$ for all finite G-sets X and such that the following axioms are satisfied:

1. For every pullback diagram of G-sets

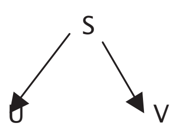


we have $M*\delta M*\gamma = M*\beta M*(\alpha)$.

2. For every pair of finite G-sets X and Y, we have $M(X) \oplus M(Y) \to M(X+Y)$ is an isomorphism.

Now we will define the $Spn(\mathcal{E})$ category (Benabou, 1967) to develop the properties of Mackey functors. Let Ɛ be a finitely compact closed category. We define the compact closed category $Spn(\mathcal{E})$ of spans in a finitely complete category Ɛ. The objects of $Spn(\mathcal{E})$ are the objects of the category Ɛ and morphisms $U \to V$ are the isomorphisms classes of spans from U to V in the bicategory of spans in Ɛ. A span from $U \to V$ is a diagram of two morphisms with a common domain S as in the diagram:

(S_1,S,S_2):



The category $Spn(\mathcal{E})$ becomes a monoidal category using the cartesian product in Ɛ as follows:

$$Spn(\mathcal{E}) \times Spn(\mathcal{E}) \to Spn(\mathcal{E})$$

defined by $U,V \mapsto U \times V$.

A Mackey functor M from Ɛ to the category Mod_k of k-modules consists of two functors $M_*:\mathcal{E} \to Mod_k$ and $M^*:\mathcal{E}^{op} \to Mod_k$ which coincide on objects and satisfy a couple of conditions. A morphism $\theta: M \to N$ of Mackey functors M and N is a family of morphisms $\theta_U: M(U) \to N(U)$ for each $U \in \mathcal{E}$ which gives two natural transformations $\theta_*: M_* \to N_*$ and $\theta^*: M^* \to N^*$. We denote the category of Mackey functors from $\mathcal{E}$ to $Modk$ by **Mky**$(\mathcal{E}, Mod_k)$ or simply **Mky** when $\mathcal{E}$ and k are understand. Now we will give an important theorem which comes from the findings of H. Lindner (Lindner, 1976).

Theorem:

The category **Mky**$(\mathcal{E}, Mod_k)$ of Mackey functors from a lextensive category Ɛ to the category Mod_k of k-modules is equivalent to the category $[Spn(\mathcal{E}), Mod_k]+$ of coproduct preserving functors.

Proof:

Here we give a sketch proof for the above theorem. Let M be a Mackey functor from from Ɛ to Mod_k. Then we have a pair (M^*, M_*) such that $M_*:\mathcal{E} \to Mod_k$, $M^*:\mathcal{E}^{op} \to Mod_k$ and $M(U) = M_*(U) = M^*(U)$.

Now define a functor $M: Spn(\mathcal{E}) \to Mod_k$ by $M(U) = M_*(U) = M^*(U)$ and $M(s_1, S, s_2)$ is given by $M^*(S_1): M(U) \to M(S)$ and $M_*(S_2): M(S) \to M(V)$. We can show that M is well defined and becomes a functors. This functor will satisfies the conditions for a Mackey functor.

Concersely, let $M: Spn(\mathcal{E}) \to Mod_k$ be a functor. Then we can define two functors M_* and M^* by referring the following diagrams:

$(\ )_*: \varepsilon \to Spn(\varepsilon)$, $(\ )^*: \varepsilon^{op} \to Spn(\varepsilon)$, and $M: Spn(\mathcal{E}) \to Mod_k$ by putting $M_*: M \circ (-)_*$ and $M^*: M \circ (-)^*$. Then we can show the necessary conditions easily. This completes the proof of the theorem.

2. Tensor Product and Closed Structure of Mackey Functors

Now we define a tensor product for the category **Mky** of Mackey functors from the category Ɛ to the category Mod_k of k-modules. This tensor produce should be equivalent to the tensor product defined on the category $[Spn(\mathcal{E}), Mod_k]+$ of coproduct preserving functors from $Spn(\mathcal{E})$ to Mod_k of k-modules by the above theorem.

The tensor product can be established as follows:

$$(M*N)(Z) = \int^{Y} M(Z \otimes Y^*) \otimes_k N(Y).$$

The Burnside functor J can be defined to be the Mackey functor on $\mathcal{E}$ such that an object U of $\mathcal{E}$ is a free k-modules. on $\mathcal{E}(1,U)$. The Burnside functor becomes the unit for the tensor product on the category of Mackey functors from the category $\mathcal{E}$ to the category Mod_k of k-modules. This convolution satisfies the necessary commutative and associative conditions for a symmetric monoidal category (Day,1970).

The closed structure for the category **Mkv** of Mackey functors from the category Ɛ to the category Mod_k of k-modules can be established using the Hom Mackey fucntor. The Hom Mackey functor is given by

$$\mathrm{Hom}(M,N)(V)=\mathbf{Mky}(M(V*\otimes -),N).$$

There is also another expression for this Hom Mackey functor, which is given by

$$\mathrm{Hom}(M,N)(V)=\mathbf{Mky}(M,\ N(V\otimes -)).$$

Therefore, the category **Mky** of Mackey functors from the category Ɛ to the category Mod_k of k-modules becomes a symmetric monoidal closed category with the above established properties. We will now investigate the monids of the monoidal category **Mky**.

3. Green Functors

A Green functor A on Ɛ is a Mackey functor , that is $A{:}\mathcal{E}\to Modk$ is a coproduct preserving functor, which equipped with a monoidal structure

$$\mu{:}A(U)\otimes_k A(V)\to A(U\otimes V)$$

and a morphism

$$\eta{:}k\to A(1).$$

Green functors are precisely become the monoids in the monoidal category **Mky.** The Burnside functor and the Hom functor $\mathrm{Hom}(A,A)$ for any Mackey functor A are also becomes monoids in **Mky** and so becomes Green functors.

Now we will describe a construction method to obtain a new Mackey functor from already known a Mackey functor. This construction method is called Dress Construction.

4. Dress Construction

The Dress construction provides a family of endofuctors $D(Y,-)$ of the category **Mky.** The Mackey functor defined as the composite $-\otimes Y$: $\mathcal{E}\to\mathcal{E}$ and $M{:}\mathcal{E}\to Mod_k$. This composite is defined by $M_Y(U)=M(U\otimes Y)$. So the Dress construction shall be defined as

$$D{:}\mathcal{E}\otimes\mathbf{Mky}\to\mathbf{Mky}$$

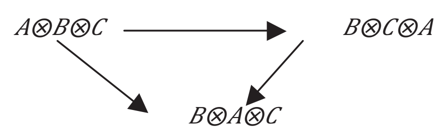
with $D(Y,M)=M_Y$. The category $\mathcal{E}\otimes$**Mky** becomes a monidal category by the following tensor product $(X,M)\otimes(Y,N)=(X\otimes Y,M*N)$.

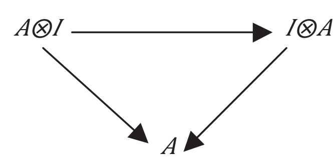
Theorem:
The Dress construction $D{:}\mathcal{E}\otimes\mathbf{Mky}\to\mathbf{Mky}$ is a strong monoidal functor.

Proof:
We need to show that $M_X*M_Y=(M*N)_{X\otimes Y}$. This can be shown by the convolution calculation and will get $(M_X*M_Y)(Z)=(M*N)(Z\otimes X\otimes Y)=(M*N)_{X\otimes Y(Z)}$. Also $D(I,J)=J$ is clear.

5. Center of a Mackey Functor

Now we define the lax centre and centre for a promonidal category $\mathcal{E}$. The lax centre $\mathcal{Z}_l(\mathcal{E})$ of $\mathcal{E}$ is defined to have objects (A,u), where A is an object of $\mathcal{E}$ and u is a natural family of morphisms $u_B{:}A\otimes B\to B\otimes A$ such that the following two diangrams commutes





Morphisms of $\mathcal{Z}_l(\mathcal{E})$ are morphisms in $\mathcal{E}$ compatible with the morphism u. We can define the tensor product in $\mathcal{Z}^l(\mathcal{E})$ as follows $(A,u)\otimes(A,v)=(A\otimes B,w)$.

The object of the centre $\mathcal{Z}(\mathcal{E})$ of $\mathcal{E}$ is an object (A,u) of $\mathcal{Z}_l(\mathcal{E})$ in which the morphism u is invertible.

Now we will consider the category $\mathcal{E}/G_c$ of crossed G-sets. It was shown by Freyd-Yetter (Freyd-Yetter, 1989) that the category $\mathcal{E}/G_c$ is a braided monoidal category. The objects are pairs $(X,|\,|)$ where X is a G-set and $|\,|{:}X\to G_c$ is a G-set morphism . The tensor product of this category $\mathcal{E}/G_c$ can be defined by

$$(X,\|)\otimes(Y,\|)=(X\otimes Y,\ \|\ \|)$$

where $\|(x,y)\|=|\,x\|y|$.

Theorem
The centre $\mathcal{Z}(\mathcal{E})$ of the category $\mathcal{E}$ of finitely compact closed category is equivalent to the category $\mathcal{E}/G_c$ of crossed G-sets.

Proof:

We have faithful a functor $\mathcal{Z}(\mathcal{E})\rightarrow \mathcal{Z}_l(\mathcal{E})$ and then we get $\mathcal{Z}(\mathcal{E})\rightarrow \mathcal{E}/G_c$. Now we let $|\ |:A\rightarrow G_c$ be an object of the category of crossed G-sets $\mathcal{E}/G_c$. Then the corresponding object of the category $\mathcal{Z}_l(\mathcal{E})$ is (A,u) where

$$U_X:A\times X\rightarrow X\times A$$

we can easily show that u is invertible. This proves the theorem.

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Modelling Colombo Consumer Price Index (CCPI) Using ARIMA Model

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Abstract: The monthly Colombo Consumer Price Index (CCPI) is the most widely used tool for measuring changes in the inflation which are important in formulating economic policies and making investment decisions. However, no methodology has been developed so far to predict CCPI at least one month ahead in Sri Lanka. Using monthly series of CCPI from January 2003 to January 2010, ARIMA (1, 2, 1) model was identified as the best fitted model for the CCPI series. The error series of the fitted model was found to be a white noise process. The model was also tested to an independent data set using CCPI from February 2010 to May 2011.

Keywords: ACF, ARIMA, CCPI, Modelling

Introduction

The Consumer Price Index (CPI) is an indicator to measure the average change in the prices paid by consumers for a specific basket of goods and services over time in a country. This "shopping basket" represents a different items consist of common consumer goods and common services purchased by the average household. The weights for each item in the shopping basket are determined based by the amount spent on these items by households in a given country.

In Sri Lanka, the CPI has been named as Colombo Consumer Price Index (CCPI) since 1953 taking the base year as 1952. The CCPI is used for deflation of current value aggregates in national accounts, formulation of policy for the determination and evaluation of wages and other monetary incomes, indexation of wages, salaries and social security benefits and social analysis. As there is a larger variation of the cost of the items selected for the shopping basket, it is obvious that this index has to be updated both spatially and temporarily. However, no attempt was taken into consideration of spatial variability.

Nevertheless, the alternative Colombo Consumer Price Index was introduced by the Department of Census and Statistics (DCS), in 2007 which is based on the Household Income and Expenditure survey (HIES) conducted by the DCS in 2002. The base year was taken as 2002 and the weighting pattern of the new index was decided based on this 2002. This new CCPI index has been denoted as CCPI (N).

The coverage of price collection for CCPI (N) has been widened to 12 centers namely Pettah, Maradana, Wellawatte, Dematagoda, Grandpass, Borella, Kirulapone, Dehiwala, Kotte, Nugegoda, Kolonnawa and Ratmalana . The weighing factors will be revised at five yearly intervals as such surveys are not conducted at monthly or annually.

The CCPI (N) is calculated using 10 major groups such as (i) Food and Non-Alcoholic Beverages, (ii) Clothing and Footwear, (iii) Housing, Water, Electricity, Gas and Other Fuels, (iv) Furnishing, House hold Equipment and Routine Maintenance of the House, (v) Health, (vi) Transport, (vii) Communication, (viii) Recreation and Culture, (ix) Education and (x) Miscellaneous and Services based on year 2002 weights.

Forecasting and modelling are important role in economic activities. The forecasting is usually carried out in order to provide an aid to decision making and planning the future. Forecasting of CCPI and its turning points are important inputs for government, businesses sector, policy makers, investors, workers and various individuals for various applications. It is obvious that this index has to be updated both spatially and temporarily. However, it can be assumed that temporal variation is more important from micro economic point of view, as policy can early be applied to temporal scale rather than spatial scale.

Though CCPI, Gross Domestic Product deflator and Wholesale price index are indicators that reflect the inflation in Sri Lanka, CCPI is the most useful indicator to reflect inflation (Korale, 2001). Most of the developed countries also use CPI as a good statistical indicator to reflect inflation. Thus the advance knowledge of inflation would be immense useful various planning and decisions making in particularly for monetary policy regimes (Gomez, 2006). However at present there is no way to detect the CCPI value in advance.

Materials and Methods

Data used

The monthly CCPI data from January 2003 to January 2010 was used to develop the model. Data from February 2010 to May 2011 was used to validate the model. All data was acquired from Department of Census and Statistics, Colombo.

Statistical Analysis

Time series approach is very suitable for describing this kind of stochastic process and also easy to establish the forecasting model to forecast CCPI. Therefore Box-Jenkins (1976) ARIMA modeling approach was used in developing forecasting model. This approach consists with four main stages: (i) make the series stationary, (ii) use of autocorrelation function (ACF) and partial autocorrelation function (PACF) of the stationary series to postulate some models, (iii) diagnostic of models and (iv) forecast.

ARIMA model

Auto Regressive Integrated Moving Average (ARIMA) is the most general class of models for forecasting a time series. Different series appearing in the forecasting equations are called "Auto-Regressive" process. Appearance of lags of the forecast errors in the model is called "moving average" process. The ARIMA model is denoted by ARIMA (p,d,q), where "p" stands for the order of the auto regressive process, 'd' is the order of the data stationary and 'q' is the order of the moving average process. The general form of the ARIMA (p,d,q) can be written as described by Judge, et al. (1988).

$$\Delta^d y_t = \delta + \beta_1 \Delta^d y_{t-1} + \ldots + \beta_p \Delta^d y_{t-p} + e_t - \theta_1 e_{t-1} - \ldots\ldots - \theta_q e_{t-q}$$

Where, Δd denotes differencing of order d, i.e., $\Delta y_t = y_t - y_{t-1}$, $\Delta^2 y_t = y_t - 2y_{t-1} + y_{t-2}$ and so forth, $y_{t-1}, \ldots\ldots, y_{t-p}$ are past observations $\delta, \beta_1, \ldots\ldots, \beta_p$ are parameters (constant and coefficient) to be estimated similar to regression coefficients of the Auto Regressive process of order "p" [AR(p)].

Where, et is forecast error, assumed to be independently distributed and $e_t, e_{t-1}, \ldots\ldots, e_{t-q}$ are past forecast errors, $\theta_1, \ldots\ldots, \theta_q$ are moving average (MA) coefficient that needs to be estimated.

Augmented Dickey Fuller (ADF) Test

The above test developed by Dickey and Fuller (1981) is used to test whether the time series process has a unit root. Thus the null hypothesis tested by DF test is:

H0: The series has unit root

H1: The series is stationary

The testing procedure for the ADF test is the same as for the but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \beta_1 y_{t-1} + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \ldots\ldots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \quad (2)$$

Where is a constant, the coefficient on a time trend and p the lag order of the autoregressive process.

By including lags of the order p the ADF formulation allows for higher-order autoregressive processes. This means that the lag length p has to be determined when applying the test.

Model adequacy measures

Akaike Information Criteria (AIC) and Bayesian Information Criterion (BIC)

The AIC and BIC (SIC) can be computed using the following formula.

$$AIC = \log\left(\frac{rss}{n}\right) + \left(\log(n) * \frac{k}{n}\right) \quad (3)$$

$$BIC = \log\left(\frac{rss}{n}\right) + \left(2 * \frac{k}{n}\right) \quad (4)$$

where;
k = number of coefficient estimated, rss = residual sum of square, n = number of observation

Results and Discussions

Temporal variability of CCPI

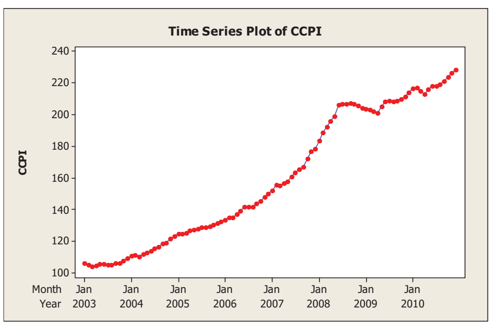

Figure 1: Time series plot of CCPI (N) [Jan 2003-Jan 2010]

From the figure 1, it can easily be seen that CCPI has been increasing over time and thus variance is increasing with time. Thus it is obvious the series is not stationary.

ACF of CCPI Series

The plot of autocorrelation (Figure 2) shows that most of the autocorrelations are not significantly different from zero, indicating the series has a long term memory. The results of the ADF test shown in table 1 confirm that series is not stationary.

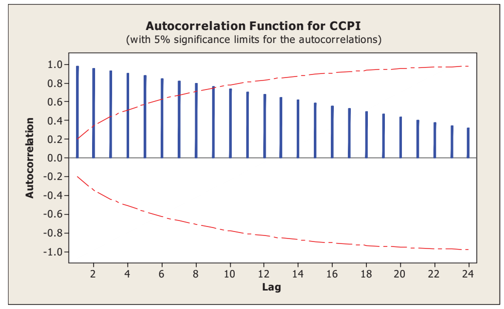

Figure 2: ACF plot of CCPI (N)

Table 1: Results of the unit root test for CCPI (N)

ADF Test Statistic		t-Statistics	Prob.
		-2.152	**0.509**
Test Critical Values	1% Level	-4.071	
	5% Level	-3.464	

Table 1 indicates that the null hypothesis that the series in levels contain unit root could not be rejected for CCPI (N). [p=0.509]. Therefore, CCPI (N) series is non-stationary. Thus, the CCPI (N) series need to be differenced to obtain a stationary series. The process is continued until a stationary series to be found. The results of ADF test for the first differenced series is shown in table 2.

Table 2: Results of the unit root test for first difference of CCPI (N)

ADF Test Statistic		t-Statistics	Prob.
		-5.377	**0.000**
Test Critical Values	1% Level	-4.072	
	5% Level	-3.465	

Table 2 indicates that the null hypothesis is rejected for the first differences of the CCPI (N). [p=0.000]. Therefore CCPI (N) series is stationary at its first difference. The plot of first lagged difference from the original data time series, which is a stationary series, is shown in figure 3.

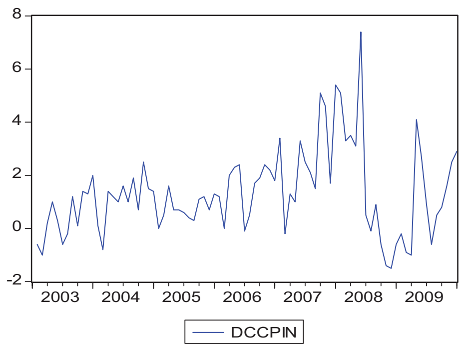

Figure 3: Time series plot of the first order difference of CCPI (N)

Figure 3 indicates that the difference with respect to the first order for CCPI (N) series is stationary but some of the price values are not located around mean of zero. Thus the second order difference for CCPI(N) series was tested for stationary using the same unit root test and the corresponding results are tabulated in table 3.

Table 3: Results of unit root test for second difference of CCPI (N)

ADF Test Statistic		t-Statistics	Prob.
		-12.643	0.000
Test Critical Values	1% Level	-4.074	
	5% Level	-3.466	

Table 3 indicates that the null hypothesis is rejected for the second differences of the CCPI (N). [p=0.000]. Therefore CCPI (N) series is stationary at its second difference. The plot of second lagged difference from the original data time series is depicted figure 4.

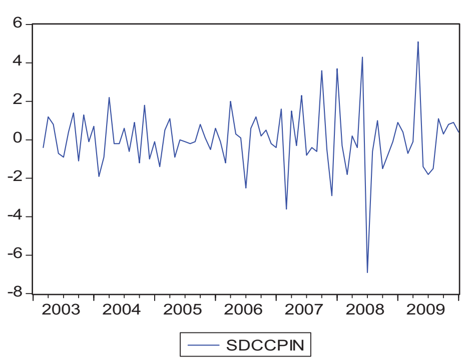

Figure 4: Time series plot of the second order difference of CCPI (N)

Figure 4 indicates that the difference with respect to the second order for CCPI (N) series is stationary because most of the price values are located around mean of zero. However, there are some spikes in the figure, representing volatility periods. However, though the series is deviated from normality ARIMA model can be fitted. As the stationary was achieved at the second difference it is required to search models of the family of ARIMA (p,2,q), where p and q are possible order of AR and MA parts.

Identification of p and q for ARIMA (p, 2, q)

The plot of sample ACF and PACF of second difference CCPI (N) series (Figure 5) was considered for identification of suitable values for p and q.

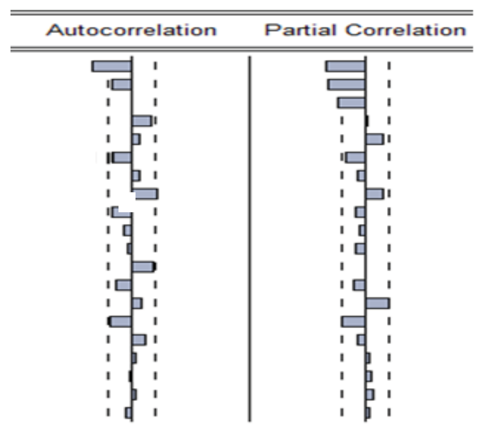

Figure 5: Sample ACF and PACF of the second order difference series of CCPI (N).

According to the figure 5, the sample ACF (SACF) has one significant autocorrelation at lag 1 and sample PACF (SPACF) has significant coefficients at lag 1, lag 2, and lag 3. Thus it can be hypothesized in the ARMA model to be fitted MA order to be 1 and AR order to be less than or equal to 3. Based on the above, the following models were considered as possible models ('parsimonious models') to represent the original series. They are: (i) ARIMA(1,2,1), (ii)ARIMA(1,2,0), (iii)ARIMA(0,2,1) and (iv) ARIMA(2,2,0).

Comparisons of parameter estimation of the selected models

Table 4: ARIMA models for D (CCPI (N), 2)

Models	Parameter Estimates	P-Value	AIC , SIC	Log likelihood	DW
ARIMA (1,2,1)	AR(1)= 0.473 MA(1)= -0.978	0.000 0.000	3.568, 3.627	-144.28	2.09
ARIMA (1,2,0)	AR(1)= -0.337	0.002	3.712, 3.741	-151.19	2.20
ARIMA (0,2,1)	MA(1)= -0.600	0.000	3.575, 3.634	-147.35	1.86
ARIMA (2,2,0)	AR(1)= -0.444 AR(2)= -0.320	0.000 0.004	3.635, 3.694	-145.22	2.15

Table 4 indicates that the coefficients of all AR (1), AR (2) and MA (1) items in the four models are significant at 5% significance level (p-value < 0.05). Hence, these four models can be considered when selecting the best fitted model from the point of view of parameter significance. A result in table 4 also indicates that of the four models the maximum log likelihood estimate and the lowest AIC and SIC values were obtained by ARIMA (1, 2, 1) model. Thus it can be concluded the best model out of the four is ARIMA (1,2,1). It should be noted that the constant term was not significant in all four models and thus model without constant term was considered.

Validation of Assumptions of residual series for ARIMA (1, 2, 1) Model

Randomness

The ACF plot of the residuals of selected model (Figure 6) shows that the residuals of the ACF are relatively small and not statistically significant. Therefore, it can be considered that the residual of the fitted model is randomly distributed.

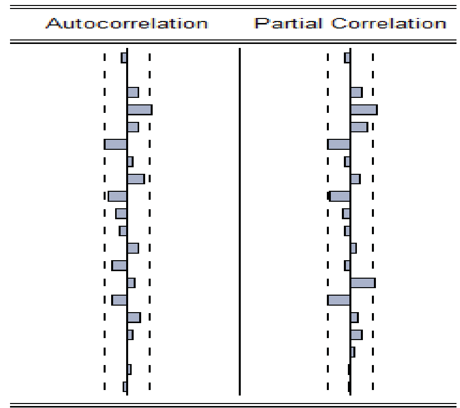

Figure 6: Plot of ACF of residuals for ARIMA (1, 2, 1)

Normality

In order to check the normality of the error series, normal probability plot of residuals was carried out (Figure 7).

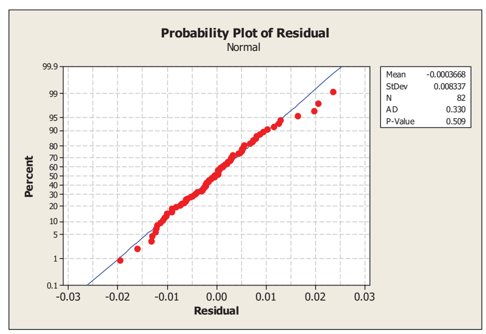

Figure 7: Normal probability plot for the residuals

Figure 7 shows that the respective Anderson-Darling statistic (AD=0.330, p=0.509) and thus it is confirmed the residual series is normally distributed.

Based on the above detailed analysis of residuals, it can be confirmed that the fitted ARIMA (1,2,1) model satisfies all the diagnostic tests. Hence, the ARIMA (1,2,1) is considered as the best fitted ARIMA model for the CCPI (N) data.

Thus, the model equation can be formed as:

$$Y_t = 2.473 * Y_{t-1} - 1.946 * Y_{t-2} + 0.473 * Y_{t-3} - 0.978 \; e_{t-1}$$

Forecasting CCPI (N) using ARIMA (1, 2, 1) Model

Before forecasting the values, it is useful to validate the present model with observed data (training set) as well as an independent data set (validation set). For the validation purpose 16 months observed CCPI (N) data from February 2010 to May 2011 (n=16) is used for validation. The forecasts errors of training and validation sets of observed data are shown in table *5*.

Table 5: Results of forecast performances statistics of ARIMA (1, 2, 1) model

Type of data	Period	Range of % error	MAPE
Training set	Jan 2003 –Jan 2010	4.39	0.63
Validation set	Feb 2010 –May 2011	3.36	0.59

Table 5 indicates that the range of percentage error and MAPE for validation set from ARIMA (1, 2, 1) model deviates from the observed data are 3.36 % and 0.59 % respectively, which is definitely be regarded as within the acceptable range.

Conclusions and Recommendations

This study aimed to modelling CCPI in Sri Lanka using ARIMA model. The time series data is not stationary at level. By applying the ADF test for the series of the second order differences we observed that the series becomes stationary, so the initial series of the monthly CCPI is integrated by second order. Then various ARIMA models were estimated by using Box-Jenkins approach. The comparative performance of these ARIMA models have checked and verified by using the accuracy statistics (AIC and SIC). The comparison indicates that the ARIMA (1, 2, 1) model as the best model and performs much better than the rest of the estimated models.

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