Analysing the volatility of all share price index using ARCH family models

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Introduction

The main objective of this research study is to analyze the volatility of all share price index using the ARCH family models.

Volatility is one of the forecast important concepts within the whole of finance. Generally, individuals tend to think about volatility as a sign of market disruption whereby securities aren't being priced fairly and therefore the capital market isn't functioning evidently. This study objective is modeling volatility of Colombo stock exchange ASPI daily data. ARCH family models are used for modeling observed statistics. Developed four ARCH family models and we were found that the best and appropriate model is GARCH (1, 1) model. It can be used to model the volatility of ASPI and can get some important decisions about changing of the stock market index.

ARCH family models are frequently used for modeling the volatility of stock markets. Most of the articles in this area of the literature deal with the analysis of the price index volatility or with the forecast of the price index.

Forecasting and volatility modeling is not very common for the Sri Lankan concept. It is a remarkable attempt to model volatility with a strong focus on Sri Lanka. GARCH (1, 1) model was identified as the best model for measuring the volatility of the ASPI return series [2]. Inflation and interest rate are the two significantly influencing macroeconomic factors on the stock market volatility of the emerging economy of Sri Lanka [1].

Methodology

1. Data: The observation period goes from 1st of January 2015 to 21st of May 2021. 1500 observations are obtained from Colombo Stock Exchange, price index daily data. Then obtained natural logarithm of ASPI. Return of stock market index has been computed using the log difference of price, that is $r_t = ln(P_t) - ln(P_{t-1})$.

2. ARCH (q) model. The ARCH method for modeling volatility has been introduced by Engle.

$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \quad u_t \mid \Omega_t \sim iid\ (0, \sigma_t^2)$

Where, $\omega > 0$, $\alpha_i \geq 0$, $i = 1,2, \ldots, q$

3. GARCH (1, 1) model. Bolerslev introduced a more general structure in which the variance model looks more like an ARMA than an AR and called this a GARCH process.

$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2 + \beta u_{t-1}^2 \quad u_t \mid \Omega_t \sim iid\ (0, \sigma_t^2)$

Where, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$

4. EGARCH (1, 1) model. Nelson proposed the EGARCH process and capture the leverage effect of stocks on the financial market. The leverage effect is exponential rather than quadratic. This ensures that the estimates are non-negative.

$Log\ (\sigma_t^2) = \omega + \alpha\frac{|\varepsilon_{t-1}|}{|\sigma_{t-1}|} + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\ (\sigma_{t-1}^2)$

Where, ω = constant, α = ARCH effect, γ = leverage parameter, β = GARCH effect.

5. TGARCH (1, 1) model. The threshold GARCH model was introduced by Zakoin and Glosten, Jaganathan and Runkle proposed the TGARCH process for asymmetric volatility structure.

$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma D_{t-1} u_{t-1}^2 + \beta\sigma_{t-1}^2$ Where, γ = leverage term, $\gamma > 0$ – asymmetry, $\gamma = 0$ – symmetry, $D = \begin{cases} 1, u_{t-1} < 0 \\ 0, u_{t-1} \geq 0 \end{cases}$.

Results and Discussion

1. Descriptive statistics. The basic analysis of ASPI and time series plot are shown in table 1 and figure 1.

From figure 1, it can be easily seen that ASPI data has been decreasing and increasing over time. Thus, it is obvious that the series is non-stationary.

Table 1. Descriptive statistics of ASPI data.

Statistical Measures	Values
Mean	6339.427
Median	6373.960
Maximum	8812.010
Minimum	4247.950
Standard Deviation	639.1977
Skewness	0.030129
Kurtosis	3.509746
Jarque-Bera	16.46702
Probability	0.000266
Observations	1500.00

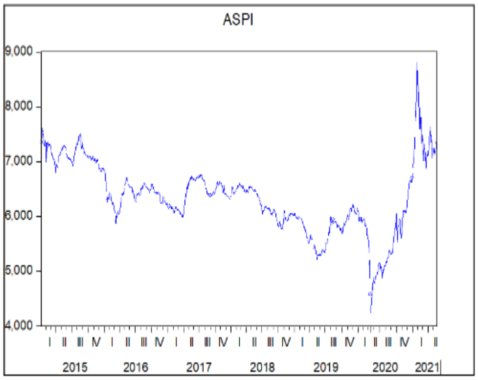

Figure 1. Time series plot for ASPI over the time.

2. Unit root test and volatility clustering

Table 2. Results of unit root test at level.

Null Hypothesis: ASPI has a unit root
Exogenous: Constant
Lag Length: 7 (Automatic - based on SIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.401618	**0.1414**
Test critical values:	1% level	-3.434526	
	5% level	-2.863271	
	10% level	-2.567740	

Table 3. Results of unit root at 1st difference.

Null Hypothesis: D(ASPI) has a unit root
Exogenous: Constant
Lag Length: 6 (Automatic - based on SIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-13.15039	**0.0000**
Test critical values:	1% level	-3.434526	
	5% level	-2.863271	
	10% level	-2.567740	

Table 2 represents that, at 5% significance level can be concluded that the ASPI series is non-stationary (P=0.1414) then checked unit root test for 1st difference. Table 3 indicate that the ASPI series is stationary at 1st difference (P=0.000). Then, obtained returns of the ASPI series and its stationary at level.

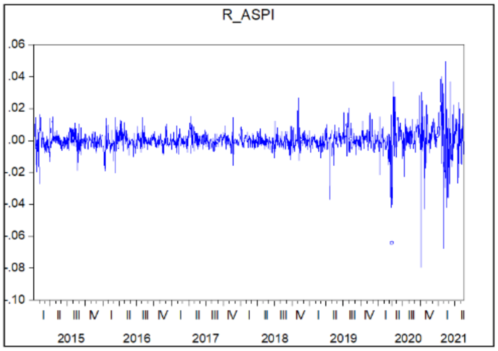

Figure 2. Time series plot for ASPI series.

From figure 2, it can be seen that there is evidence volatility clustering exists for our series. Then obtained AR (1) model using the least squared method and checked its residuals, which can be represented below.

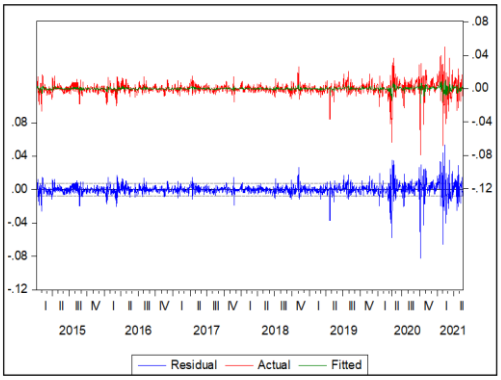

Figure 3. Residuals of the model.

Looking at the figures 2 to 3 volatility clustering periods when large changes are followed by further large changes and periods when small changes are followed by further small changes. When these things happen for residuals clearly can be said that volatility clustering exists. In order to be sure, can be run a heteroskedasticity test.

3. Testing ARCH effect. Table 4 indicates that, the null hypothesis is rejected and it can be concluded there is an ARCH effect exist (Obs*R-squared 69.6652 and P=0.00).

According to the above results, both conditions of volatile clustering and ARCH affect existing and satisfied. Therefore, all the justifications are satisfied to run ARCH family models.

Table 4. ARCH test.

Heteroskedasticity Test: ARCH			
F-statistic	72.96776	Prob. F(1,1495)	0.0000
Obs*R-squared	**69.66517**	**Prob. Chi-Square(1)**	**0.0000**

4. ARCH type model analysis. Four ARCH types of models were developed. They are the ARCH (5) model, GARCH (1, 1) model, EGARCH (1,1) model, and TGARCH (model). Estimated models are given in below tables 5 to 8.

Table 5. ASPI (5) model.

Dependent Variable: R_ASPI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2 + C(7)*RESID(-4)^2 + C(8)*RESID(-5)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	**-0.000252**	0.000123	-2.056268	**0.0398**
R_ASPI(-1)	**0.237088**	0.028625	8.282432	**0.0000**
Variance Equation				
C	**1.06E-05**	3.81E-07	27.85955	**0.0000**
RESID(-1)^2	**0.194886**	0.031746	6.138984	**0.0000**
RESID(-2)^2	**0.194010**	0.029069	6.674029	**0.0000**
RESID(-3)^2	**0.070729**	0.024211	2.921424	**0.0035**
RESID(-4)^2	**0.231699**	0.020115	11.51896	**0.0000**
RESID(-5)^2	**0.168360**	0.014632	11.50670	**0.0000**

Table 6. GARCH (1, 1) model.

Dependent Variable: R_ASPI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	**-0.000302**	0.000123	-2.456862	**0.0140**
R_ASPI(-1)	**0.215727**	0.030792	7.006022	**0.0000**
Variance Equation				
C	**1.41E-06**	2.21E-07	6.386157	**0.0000**
RESID(-1)^2	**0.195984**	0.012167	16.10722	**0.0000**
GARCH(-1)	**0.790513**	0.013844	57.10353	**0.0000**

Table 7. EGARCH (1, 1) model.

Dependent Variable: R_ASPI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	**-0.000428**	0.000109	-3.913899	**0.0001**
R_ASPI(-1)	**0.183155**	0.027204	6.732687	**0.0000**
Variance Equation				
C(3)	**-0.583721**	0.056208	-10.38505	**0.0000**
C(4)	**0.331974**	0.017202	19.29818	**0.0000**
C(5)	**0.011895**	0.010345	1.149861	**0.2502**
C(6)	**0.966696**	0.004734	204.2182	**0.0000**

Table 8. TGARCH (1, 1) model.

Dependent Variable: R_ASPI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	**-0.000282**	0.000131	-2.156210	**0.0311**
R_ASPI(-1)	**0.214034**	0.030705	6.970573	**0.0000**
Variance Equation				
C	**1.42E-06**	2.21E-07	6.414842	**0.0000**
RESID(-1)^2	**0.204686**	0.013874	14.75299	**0.0000**
RESID(-1)^2*(RESID(-1)<0)	**-0.021142**	0.021588	-0.979350	**0.3274**
GARCH(-1)	**0.790907**	0.013940	56.73764	**0.0000**

5. Model selection. Compared, four models in order to find the best model using AIC, SIC, H-Q, and Log-likelihood values. The results are given in table 9.

According to the above test results AIC, SC, and H-Q are the) is a high value for GARCH (1, 1) model compared with others. Therefore, the estimated GARCH (1, 1) model is the best model for determining the volatility of ASPI.

6. Diagnostic checking for GARCH (1, 1) model. The heteroskedasticity and serial correlation of residuals were tested (Table 10).

Table 10 indicates that Obs*R-squared is not significant (P-value - 0.9999) at 5% significance level. Therefore, the hypothesis of no ARCH effect cannot be rejected. Hence, there is no ARCH effect in the residuals.

Table 9. Model selection results.

Model selection criteria	ARCH (5) model	GARCH (1, 1) model	EGARCH (1, 1) model	TGARCH (1, 1) model
AIC	-7.521902	**-7.552830**	-7.542373	-7.551803
SIC	-7.493534	**-7.535100**	-7.521097	-7.530527
H-Q	-7.511333	**-7.546224**	-7.534446	-7.543876
Log likelihood	5641.904	**5662.069**	5655.237	5662.300

Table 10. Results of heteroskedasticity test.

Heteroskedasticity Test: ARCH

F-statistic	9.80E-09	Prob. F(1,1495)	0.9999
Obs*R-squared	**9.82E-09**	**Prob. Chi-Square(1)**	**0.9999**

Table 11. Correlogram for sample ACF and PACF of squared residuals.

Date: 06/10/21 Time: 23:58
Sample: 1/02/2015 5/21/2021
Included observations: 1498

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.000	0.000	1.E-08	1.000
		2	-0.015	-0.015	0.3396	0.844
		3	-0.012	-0.012	0.5631	0.905
		4	0.039	0.039	2.8811	0.578
		5	-0.006	-0.007	2.9425	0.709
		6	-0.010	-0.009	3.0793	0.799
		7	-0.016	-0.015	3.4464	0.841
		8	-0.001	-0.003	3.4493	0.903
		9	0.003	0.002	3.4591	0.943
		10	0.007	0.008	3.5422	0.966
		11	0.002	0.003	3.5458	0.981
		12	0.016	0.016	3.9323	0.985
		13	-0.010	-0.010	4.0803	0.990
		14	0.017	0.016	4.5017	0.992
		15	0.003	0.003	4.5135	0.996
		16	0.005	0.004	4.5540	0.998
		17	-0.000	0.002	4.5541	0.999
		18	-0.004	-0.004	4.5738	0.999
		19	-0.012	-0.012	4.7935	1.000
		20	-0.016	-0.016	5.1637	1.000
		21	-0.020	-0.020	5.7864	1.000
		22	-0.010	-0.010	5.9365	1.000
		23	-0.020	-0.020	6.5422	1.000
		24	-0.010	-0.011	6.7082	1.000
		25	-0.006	-0.005	6.7567	1.000
		26	0.003	0.001	6.7690	1.000
		27	0.006	0.006	6.8166	1.000
		28	-0.014	-0.015	7.1116	1.000
		29	-0.024	-0.024	7.9630	1.000
		30	0.001	-0.000	7.9638	1.000
		31	0.005	0.004	8.0096	1.000
		32	0.032	0.033	9.5813	1.000
		33	0.021	0.024	10.257	1.000
		34	-0.006	-0.004	10.309	1.000
		35	-0.014	-0.013	10.626	1.000
		36	-0.005	-0.008	10.669	1.000

The table 11 indicates that, all P-values of autocorrelations are not statistically significant at 5% significance level. Therefore, we can't reject null hypothesis and statistically can be concluded that residuals are not serially correlated.

Based on the above analysis of residuals confirmed that GARCH (1, 1) model is the best and appropriate model.

Conclusion

This study mainly focused on modeling the volatility of ASPI using ARCH family models. In the analysis, it was found that ASPI data is stationary at 1st difference. Four ARCH family models were estimated for the data. Among these four models, based on the model selection and diagnostic criteria it was conformed that GARCH (1, 1) model is the best model for ASPI in Colombo stock exchange.

References

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