Chebyshev polynomial approximation to solutions of third order linear differential equations

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Differential equations are used in variety of fields including pure and applied mathematics, engineering and physics. Many of these fields are concerned with the properties of different forms of differential equations. Solving differential equations along with certain conditions, called initial value problem (IVP) or boundary value problem, become very important in many research situations. Differential equations raised in real-world problems are not always explicitly solvable. That is, they do not always have closed from solutions. Instead, numerical methods can be used to approximate solutions. Chebyshev polynomials are two sequence of polynomials related to the sine and cosine functions. They are orthogonal polynomials that are related to De Moivre's formula. They have numerous properties which make them useful in areas like solving polynomials and approximating functions. Chebyshev approximation produces a nearly optimal approximation, coming close to minimizing the absolute error. Robertson, A. S. (2013) discussed a method for finding approximate particular solution for second order non-homogeneous ordinary differential equations. Yang Zhongshu and Zhang Hongbo (2015), in their work, developed a computational method for solving class of fractional partial differential equations with variable coefficients based on Chebyshev polynomials. In this research we developed a method to find approximate particular solution for third order linear differential equation. Here we used Chebyshev polynomial to approximate the source function and the particular solution of an ordinary differential equation. The derivatives of each Chebyshev polynomial will be represented by linear combinations of Chebyshev polynomials. Then the differential equations will become algebraic equations. Here we took first six polynomials of Chebyshev polynomials of first kind because when we approximate the function by Chebyshev polynomials, the coefficients of higher order Chebyshev polynomials are negligible. Our main objective of this study is to approximate the solution of third order linear differential equation by Chebyshev polynomial as close as possible to the exact solution. We applied our proposed method on some algebraic, trigonometry and exponential functions. This approach is compared with another well-known existing method, Euler's method. Our proposed approach provides more efficiency compared to the existing method.