Abstract ID: P20

LINEAR STABILITY ANALYSIS OF MORE GENERAL LINEAR ITERATION SCHEME IN THE IMPLEMENTATION OF IMPLICIT RUNGE KUTTA METHODS

S. Kajanthan[a*], R. Vigneswaran[b]

[a]*Department of Interdisciplinary Studies, Faculty of Technology, University of Jaffna, Sri Lanka.*
[b]*Department of Mathematics and Statistics, Faculty of Science, University of Jaffna, Sri Lanka.*

[*]*kajanthans@univ.jfn.ac.lk*

Abstract

A more general linear iterative scheme solves non-linear equations arising in the implementation of implicit Runge-Kutta methods proposed by Cooper and Butcher is of the form

$$\left[I_s \otimes \left(I_n - h\lambda J\right)\right] E^m = \left(BS^{-1} \otimes I_n\right) D\left(Y^{m-1}\right) + \left(L \otimes I_n\right) E^m,$$
$$Y^m = Y^{m-1} + \left(S \otimes I_n\right) E^m, \quad m = 1, 2, \ldots,$$

where B and S are real $s \times s$ non-singular matrices and L is strictly lower triangular matrix of order s, and λ is a real constant. They showed that successive over relaxation technique applied to improve the convergence rate of this scheme. Later, convergence result of this scheme established by proving some theoretical results suitable for stiff problems. This article examines stability properties of this linear iterative scheme with the alternate approximation

$$y_{n+1}^k = y_n + h\left(b^T A^{-1} \otimes I\right)\left(Y^k - e \otimes y_n\right) .$$

It is better to use because it requires less evaluation of f and is more accurate for stiff problem. For a fixed starting value $Y^0 = e \otimes y_n$, we obtain

$$y_{n+1}^k = R(z) y_n + b^T A^{-1} M\left(z\right)^k \left(I - \frac{1}{z} A^{-1}\right)^{-1} e\, y_n$$

here $R(z) = 1 + zb^T\left(I - zA\right)^{-1} e$ is the stability function of the Runge-Kutta method and $M\left(z\right) = \left(Q - zT\right)^{-1} [Q - I + z\left(A - T\right)]$ where $Q = SB^{-1}\left(I - L\right)S^{-1}$ and $T = \lambda SB^{-1}S^{-1}$.
This shows that linear stability properties were preserved in the limit.

Keywords: *iteration scheme, linear stability, implicit Runge Kutta methods*